DOCUMENT RESUME

ED 197 964 SE 034 040

AUTHOR	Fitzgerald, William M.; Shroyer, Janet
TITLE	A Study of the Teaching and Learning of Growth Relationships in the 6th Grade. Final Report.
INSTITUTION	Michigan State Univ., East Lansing.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	6 Apr 79
GRANT	NSF-SED77-18545
NOTE	137p.; Not available in hard copy due to marginal legibility of original document. Appendix A missing from document prior to its being shipped to EDRS for filming.
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Educational Research; *Elementary School Mathematics; Evaluation; *Grade 6; Intermediate Grades; Learning Theories; *Mathematics Instruction; *Pilot Projects; Problem Solving; *Teaching Methods
IDENTIFIERS	*Mathematics Education Research

ABSTRACT

A mathematics unit for sixth-grade classes was developed in which each of eight activities contains a challenge conveyed in the form of a story. Each activity requires students to manipulate concrete materials such as ceramic tiles, marked string, wooden cubes, and square paper. The mathematical content consists of concepts such as area, perimeter, surface area, and volume; relationships relating to growth and variability; and measuring skills such as counting and rule-governed behavior. Teachers from different types of schools and settings were selected and trained. Four were chosen to be video-taped. Data on student understanding of materials was obtained through individual assessments completed in interviews, final evaluations accomplished through written examinations, and post evaluations administered in a one-to-one setting of student and investigator. Teachers provided descriptions of their teaching experiences and their judgments of student performance. Each activity was evaluated in terms of student and teacher performance. The report contains information on development of the units, data collection procedures, and results of the study. A list of 13 conclusions provide insight into problems encountered and possible methods for improvement. (Author)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED197964

SE

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Patricia Babb
of the NSF

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

FINAL REPORT

A STUDY OF THE TEACHING AND LEARNING OF GROWTH RELATIONSHIPS IN THE 6TH GRADE

William M. Fitzgerald, Director

Janet Shroyer, Coinvestigator

Department of Mathematics
Michigan State University
East Lansing, Michigan

This study was supported by the National Science Foundation
Contract No. SED 77-18545. Submitted April 6, 1979.

SE 034 040

ACKNOWLEDGEMENTS

We are deeply grateful for the enthusiastic cooperation and the professional contributions of the project teachers.

East Lansing Public Schools

Robert Arndt, Hannah Middle
William Gale, MacDonald Middle

Eaton Rapids Public Schools

Susan King, Union

Haslett Public Schools

Mark Straubel, Haslett Middle

Holt Public Schools

Nancie Lubbers, Holt Middle

Lansing Public Schools

John Baker, Elmhurst
Alan Donaldson, Forest View
Elaine Elliott, Northwestern
Lucy Knappen, Moores Park
Ronald Kolar, Pleasant View
Jan Pfiefer, Elmhurst
Sandy Szedlak, Mt. Hope

Okemos Public Schools

Sheila Schwartz, Kinawa Middle

The Institute for Research on Teaching, College of Education, provided the video capability for the project.

CONTENTS

	Page
Introduction - Origins of the Project	1
The Nature of the Project	2
The Mouse and Elephant Unit	3
Chronology	7
Pilot Teaching	7
Teacher Selection	8
The Training Program	9
The Teaching of the Unit by Project Teachers	9
Final Project Activities	10
Data Collection Procedures	11
Student Data	11
Individual Assessments	11
Final Evaluation	13
Post Evaluation	14
Content Analysis	16
Teacher Data	24
Teaching Performance	24
Daily Teaching Logs	24
Debriefing Sessions	25
Follow-up Questionnaire	25
Teacher Judgment of Student Performance	25
Pupil Sorts	25
Item Analysis	26
Daily Reports on Individual Students	26
Observer Data	26
Video Records	27
Classroom Observation Forms	28
Results	29
I. Student Data	29
A. Concepts	30
Student Success Rates	31
Difficulties	32
B. Relationships	34
Variability Relationships	34
Success Rates	35
Difficulties	36
Growth Relationships	39
Success Rates	40
Difficulties	41

Page

C. Skills 42
  Nature of Skills Observed 42
  Applications Across Representational Variations 45
  Success Rates 46
  Discussion About Skills 47

D. Student Performance During Activities 49

E. Student Reactions to the Unit 53

II. Teacher Data

A. Teachers' Performance of the Instructional Phases 55
  Launching Phase 56
  Discussion of Variations 58
  Exploration Phase 60
  Discussion of Variations 61
  Summarizing Phase 63
  Discussion of Timing 64

B. Teacher Performance in the Activities 68

C. Teacher Judgment of Student Performance 70
  Teacher Agreement on Student Performance Before and After the Mouse and Elephant Unit 70
  Teacher-Observer Agreement on Daily Student Progress 72
  Teacher Judgment of Student Performance on the Final Evaluation by Test Item 74
  Teacher Judgment of Individual Student Performance on Final Evaluation 77
  "Surprises" 78

D. Project Teachers Reactions to the Unit After Ten Months 80

III. Class Differences 84

IV. Discussion and Conclusions 86
  Conclusions Relating to Students 86
  Conclusions Related to Teachers 87
  Conclusions Regarding the Unit 88
  Conclusions Regarding Research Methodology 90

Appendicies 93
  Appendix A - Script of the Mouse and Elephant Unit (In separate ring-binder)

  Appendix B - A Brief Description of the Project Teachers and their Classrooms

  Appendix C - Teachers' Responses to the Script

  Appendix D - Evaluation Instruments

TABLES

Page

1. Introduction and Review of Concepts and Relationships in the Activities. 6

2. Percent of Students Demonstrating Understanding of Concepts on the Final Evaluation and Individual Assessments. 31

3. Percent of Success on Variability Relationships. 35

4. Responses of Students on Final Evaluation When Building Rectangles with a Perimeter of 20. 37

5. Responses of Students on Final Evaluation When Building Solid Blocks with Volume of 12. 39

6. Percent of Students Successful on Post Evaluation. 40

7. Rules Students Used for Measuring. 44

8. Percent of Students Using Counting or Rules on Concrete Models (I) or Pictures (II) During Individual Assessment. 45

9. Percent of Students Demonstrating Measurement Skills. 47

10. Student Reactions to the Unit. 53

11. Minutes Devoted to Phases by Activity and Teacher. 65

12. Measure of Association (Gamma) Between Teacher Judgment of Student Mathematics Performance Before and After Teaching Unit. 71

13. Measure of Association (Lambda) Between Teacher and Investigator Daily Judgment of Student Progress. 73

14. Teacher Prediction and Student Performance on the Final Evaluation. 76

15. Measures of Predictability (Somers' d) of Students Final Evaluation Score from Teachers' Pupil Sorts. 77

16. Frequencies of Teacher Reported "Surprises" by Teacher and Nature of Surprise. 79

17. Distribution of Scores on Final Evaluation Ordered by Class Means. 85

Final Report: A Study of the Teaching and Learning of Growth Relationships in the 6th Grade

Introduction - Origins of the Project

The powerful, yet simple ideas of relative growth have intrigued the human intellect since Galileo, and have been discussed by scholars such as Julian Huxley, D'arcy Thompson, and more recently, David Hawkins. Attempts were made in the early 60's to include these ideas in the new physics, new biology and new mathematics projects with little or no success.

When the investigators in this project began experimenting with the teaching and learning of the ideas associated with relative growth at a variety of age levels, we received very mixed responses. Many good teaching materials and ideas were available among the "tailings" of the new curriculum projects. Students at several age levels seemed to understand some of the ideas but their responses were very "spotty". Most middle grade teachers seemed to have had no experience with the ideas. Research on the subject was almost nonexistant.

The investigators began development of the first version of the "Mouse and Elephant" unit in late 1976 to be used in Shroyer's doctoral study on the mental life of teachers. The original unit consisted of a pack of 20 5 x 8 activity cards, one side describing challenges and suggested activities for teachers to present to their class and on the other side, general solutions and explanations of the activities for the teacher.

The unit, in the form of activity cards, was viewed as an example of a half-product of curriculum materials suggested by Kilpatrick as a possible new emphasis in the post-new-math-era.

The three teachers in that research project, in the spring of 1977, were experienced, former USMES teachers, known for their competence and were willing to be scrutinized closely. Yet, because the ideas were not familiar, and the activities novel, the investigators sensed a great frustration because of the disparity between how the unit evolved and was taught in the classrooms and the potential for the unit in the minds of the investigators. Thus, a proposal was written to NSF to study the problem in more depth.

The Nature of the Project

The investigators believed that the combination of a rich unit of mathematical content to be introduced into an active classroom of sixth grade children, using concrete manipulative materials and taught by an experienced, enthusiastic teacher would result in an optimum mathematical learning experience for the students. With the funding of the project, they set out to capture the essential elements which would be required for such an experience to occur.

While it is easy to agree that the circumstances described above are very desirable, a little reflection reveals that there is much about the situation that is not known to us, the practictioners in mathematics education at this time.

First, the ideal unit for teaching, including the content, the strategies, and the materials did not exist in any acceptable form. Second, in an ideal classroom setting, how much mathematics could we expect sixth grade students to learn? Third, where would one find the teachers, and how would they best be prepared to teach the unit?

To conduct this project, we proceeded to develop a model unit, teach it in some pilot classes, recruit some experienced teachers, provide them with some training and materials, and monitored their teaching of the unit.

The study attempts to answer the following questions:

With regard to students

1. To what extent did the students learn the mathematical content of the unit?
    a) How much did they understand the concepts and the relationships?
    b) To what extend did they acquire the measuring skills?
2. What was the nature of the student performance on the activities?
    a) What difficulties did they encounter?
    b) What variations in measuring skills were present?

With regard to teachers

3. What was the nature of the variation in how the teachers taught the unit? (I.e., how did they execute the phases of instruction?)
4. What was the nature of the teachers' reactions to the activities?
5. To what extent did teachers judgment of student learning agree with other indications of student learning?

The Mouse and Elephant Unit

The second generation of the mouse and elephant unit is a sequence of scripted activities about the measurement of area, perimeter, surface area and volume and the relationships of change during growth. It is intended to be taught to classes in grades five through eight. The unit was designed with the following premises in mind.

1. The unit encompases an important "chunk" of mathematical ideas. These ideas usually have historical significance in the development of our culture. They also frequently have a history of being difficult to teach. They include important concepts and fundamental relationships.

2. The ideas are couched in a large, interesting problem which provides the motivation for the unit.

3. The unit consists of a sequence of carefully structured activities which embed the major concepts in fantasy and require each child to be engaged in the manipulation of concrete materials.

4. The unit involved an entire class pursuing the solution of the big problem - sometimes in total class discussion and at other times working in small groups or individually. The classroom is fluid.

5. The challenges posed to the children in the activities can be pursued at different levels of sophistication to accomodate the variety of cognitive levels present in a middle grade classroom.

6. Each activity provides a collection of extra challenges to keep the faster students motivated and extended.

7. Each activity is planned and described sufficiently well so the teacher can know what will probably happen, will be able to anticipate responses, and will have available an assortment of possible alternatives.

This unit consists of eight activities which lead toward the solution of the major problems. The major problems are:

(Given the facts that an elephant is really a large mouse, and that you know their respective heights:)

A. How many mouse coats can be cut from an elephant coat?

B. How many mice are needed on a balance scale to balance an elephant?

Each of the eight activities contains a challenge for the students which is conveyed in a story. The major concepts are given fantasy interpretations which are then gradually shifted to mathematical interpretations. Thus, the number of people sitting around a banquet table becomes perimeter, etc. All of the activities require the students to manipulate concrete materials such as ceramic tiles, marked string, wooden cubes, square paper and geoblocks. Listed below are the fantasy tasks and the corresponding mathematical tasks for the eight activities.

Activity 1. Find the number of small tables needed to provide these banquet tables and how many people can sit around them. (Measure the area and perimeter of rectangles.)

Activity 2. Find all possible banquet tables which can be formed using 24 small tables. (Measure the perimeters of all rectangles with area of 24.)

Activity 3. Find all possible banquet tables which allow 24 people to be seated. (Measure the area of all rectangles with perimeter of 24.)

Activity 4. Find the cost of space armor jackets for these food pellets. (Measure the surface area of various rectangular solids, (solid blocks).)

Activity 5. Find all the possible costs of space armor for shipping 24 food pellets. (Measure the surface area of all solid blocks with volume of 24.)

Activity 6. What is the maximum number of food pellets that can be shipped for $60 or less? (Measure the volume of all solid blocks with a constant surface area.)

Activity 7. What is the area and perimeter of a 40 year-old square? How many people can sit aroung a 40 x 40 table? How many small tables are required? (Measure the area and perimeter of growing squares.)

Activity 8. What is the surface and volume of a 40 year-old cube? (Measure the surface area and volume of growing cubes.)

Conclusion. If an elephant is a 40 year-old mouse, how many mouse coats can be cut from an elephant coat and how many mice are needed to balance the elephant?

The mathematical content of this unit consists of concepts, relationships and measuring skills. The concepts are area, perimeter, surface area and volume. These are embedded in concrete models of square ceramic tiles, marked strings, square paper and wooden cubes.

Two kinds of relationships are studied. Variability relationships are those in which one measure is fixed while a related measure varies. Thus, we alternately fix and vary area and perimeter and also surface area and volume. Growth relationships are changes in linear area and volume measures as an object grows or shrinks.

Measuring skills are considered on two levels; basic counting of segments, squares, or cubes at the lower level, and some kind of rule-governed behavior at the higher level.

The concepts and relationships are introduced to the students in a systematic sequence in the activities and are reviewed in subsequent activities as shown in Table 1.

TABLE 1

Introduction and Review of Concepts and Relationships in the Activities

	Area	Perimeter	Volume	Surface Area	Variability Relations	Growth Relations
Activity 1	I	I				
2	R	R			I	
3	R	R			R	
4			I	I		
5			R	R	I	
6			R	R	R	
7	R	R				I
8			R	R		I
Conclusion	R	R	R	R	R	R

Each activity is organized into three instructional phases: launching, exploring and summarizing. Teaching roles and techniques are different in the different phases. Briefly, the phases can be described as follows:

a. launching. Using whole class instruction, the teacher introduces the activity by clarifying the necessary new concepts and reviewing old ones, ensures that the task, information and directions are understood, and issuing the major challenge.

b. exploring. While the students are pursuing the solution to the challenge in small groups or individually, the teacher moves about the room maintaining on-task behavior by assisting, correcting, prodding and offering extra challenges to those students who are ready and interested to further their understanding and knowledge.

c. summarizing. Returning to whole class instruction, the teacher elicits and displays results in an organized fashion to encourage searching for patterns and relationships. Rules which are identified can be recorded in mathematical symbols and verified with further examples.

Chronology

Official notification of the project award was received on September 30, 1977. Because of the need for permission slips, administrative approval, etc., the pilot teaching began on November 1, 1977.

Pilot Teaching

The careful development of the unit began with a pilot group of five sixth grade students in the Moores Park School in Lansing. The five students represented a wide spectrum of ability from a typical class. One investigator taught the group of five while the other video-taped the interaction. From the careful planning of the instruction and extensive analysis of the video tapes of the lessons, a script of the unit began to take shape. The sequence of questions and challenges, the typical responses, the difficulties, and the discoveries were beginning to become more common. At the completion of the work the the Moores Park Pilot group, we wrote the first version of the script for the Mouse and Elephant Unit.

Pilot classes were selected at Mt. Hope Elementary School in Lansing. Mt. Hope is a "cluster" school which means it has only kindergarten, 5th and 6th grades because of desegregation patterns imposed by Federal courts. The students were platooned for math, science and social studies.

A former USMES teacher taught three mathematics classes each morning. The project staff selected the first and third classes to use as pilot classes, each taught by one of the co-investigators beginning on November 28. The regular teacher operated the video camera and the other co-investigator served as the observer in the classroom. By teaching, observing and analyzing the video tapes, we were able to reach an acceptable approximation to a script for the unit which would be provided for the 13 participating teachers in the project.

Teacher Selection

The project teachers were recruited and selected by a variety of means. Recommendations were sought from mathematics coordinators and administrators. Several of the teachers had participated in the Lansing District USMES project in 1972-73 and were friends of the co-investigators. Attention was given to attaining a variety of characteristics in the population. The sexes were evenly divided. Schools were selected from urban, suburban and rural districts. Teachers were selected from middle schools as well as self-contained classrooms in elementary schools. One seventh grade and one fifth grade were selected along with eleven sixth grade teachers in order to provide some information about the grade boundaries.

The teachers were paid $100 to participate in the project. The four teachers who were video-taped were paid an additional $100.

The Training Program

The project teachers met with the project staff for three sessions in three consecutive weeks from 4:00 p.m. to 8:00 with a dinner break, during January, 1977. The purposes of the training program included (1) teach the project teacher the activities of the unit,. (2) explain the rationale for the approach to the unit (to the extent that it was developed at that time), (3) provide an opportunity for the project teachers to become acquainted with each other, (4) become accustomed to the use of video-taping apparatus and it's effect on the classroom.

The training program seemed much too short. We covered activities 1, 2, and 3 the first night; 4, 5 and 6 the second night and 7 and 8 the third night. In addition, we measured the teachers judgments about the anticipated student performance and gathered other demographic data. We also showed them some replay on the video-tape of the difficulty they had in activity 6 of finding the maximum volume for a fixed surface area.

Morale remained high during the training period. There was a strong feeling of professional involvement. We had the collection of teachers we wanted and we were pleased.

The Teaching of the Unit by Project Teachers

Four of the teachers were chosen to be video-taped. The selection was arbitrary. We wanted to provide diversity. We selected 2 men and 2 women, 1 self-contained classroom and 3 middle school classrooms, one seventh grade and 3 sixth grade, one urban and 3 suburban classrooms. Our rationale for selecting the four teachers were very slim and nearly any other combination would have been as justifiable.

Funds were available for the purchase of three complete sets of classroom materials so the unit was taught by three teachers at a time with one of the three being video-recorded. The teaching and evaluation of the unit required about three weeks. Therefore, the unit was being taught from February to May, 1978.

Final Project Activities

A final meeting of the project teachers and staff was held in May to summarize and reflect on the project and share ideas and concerns.

Analysis of the video tapes, the other data and the preparation of the final report was conducted between February, 1978 and March, 1979.

DATA COLLECTION PROCEDURES

The primary sources of data about the teaching and learning of the Mouse and Elephant Unit were students, teachers and observers. Clinical interviews and written examinations were used to determine the extent to which students learned the content. Teachers were asked to make judgments about their students and to keep logs on their teaching experiences. Observers were the two co-investigators They video-taped four of the classes, kept records of interesting events of teacher/student participation, and made daily judgments about students.

The following sections on student, teacher, and observer data contain descriptions of the instruments and the data gathering procedures.

I. STUDENT DATA

There were three different ways of collecting data on student understanding of the concepts, skills, and relationships contained in the unit. They were Individual Assessments (I.A.), Final Evaluations (F.E.), and Post Evaluations (P.E.).

The Final Evaluation was the only written examination and the only instrument used to test all students in the thirteen classes. Post Evaluation interviews were only conducted with students who scored 78% or more on the Final Evaluation. The Individual Assessments were interviews with students in the videoed classes only.

Individual Assessment

The Individual Assessment consisted of two subtests administered at different times during the unit. They were Rectangular Assessment (RA) and Solid Block Assessment (SA). The first dealt with measures of rectangles; the topics of Activities 1-3. The second centered on the measures of rectangular solids; the topics of Activities 4-6. The Rectangle Assessment was given any time students were available after Activity 1 up through the day in which Activity 4 was completed. The Solid Block Assessment was given after students had participated in Activity 5 but before the last activity, (Activity 8) had begun. Students did not all take the same assessment at the same exact point within the unit, but the range of testing days was necessary in order to find opportunities out of class time to conduct the interviews. The interviews were held in whatever rooms could be made available so that the investigators and students could sit down together away from other students and teachers.

The style of questioning in these clinical interviews was casual but carefully designed to elicit as much information from the students as possible without being too directive. For example, when showing a rectangle drawn on a paper for which tiles and string were available to do the measuring, students were asked what measures they could give rather than for specific measures. Thus, students were contributing maximally with the interviewer providing direction or assistance only to those students who needed it. This approach provided the investigators with an opportunity to gather many impressions about the extent to which students were able to express themselves.

To the extent allowed by the limited time for these interviews, the investigators were able to probe or assist students in order to explore how students comprehended the mathematical ideas. Students were almost always cooperative and eager to please as well as be correct. No negative feedback or results were given to students about the accuracy of their responses.

The primary purpose for administering the individual interviews was to acquire data on what students learned. However, these exchanges were also a compromise in the initial plan. Early in the preparation phase of the study stimulated recall sessions were planned to be used with students in an effort to capture their memorable learning experiences, both positive and negative.

Just the simple logistics of bringing one or more students together with an investigator for sufficient time to view a reasonable chunk of a lesson on video-tape was enough to dissuade the investigators from doing so except occasionally with small groups of students. But after some pilot efforts another even more potent effect caused the plan to be scrapped. Students in early adolescence are characteristically often intolerant of themselves and others. Showing students video-tapes of themselves and their class did little to gather any insights as to student cognitive or even affective experiences. However, individual attention, another characteristic of their developmental stage, was accepted and appreciated by the students. Thus, the importance of the clinical interview was inflated by being the only source of data about individual students beyond observations which could be made during the activities.

Final Evaluation

The Final Evaluation consisted of six problems designed to sample student understanding of content other than the growth relationships. This written examination, administered by one investigator, was given the day after the completion of the unit in each class. (The teachers didn't finish the unit on Fridays.) The regular teacher was usually not present during the final evaluation in order to avoid pressure for success and to enable the teacher to be interviewed by the other investigator.

During the test the students were allowed to raise their hands to request assistance in reading the problem. Policy also allowed for assistance with the mathematical terminology but not instruction on the meaning or methods. When requested, the students were given reminders of the story language used to convey the concepts or tasks.

The first three pages were green and the students were encouraged to use tiles, string, and cubes to help them solve the problems. The three final pages were white, one presenting a picture of a solid block and the other two providing only written descriptions including the abstract dimensions of the figures to be measured.

Prior to beginning the written examination, students were asked to write their reactions to the unit on the back of the last page. These free form responses were then categorized.

Post Evaluation

Students scoring 78% or more on the final evaluation were interviewed individually to determine the extent to which they had an understanding of the growth relationships. This evaluation was administered the first school day after the final evaluation.

The prerequisite score avoided first rating students who possessed neither the concepts nor the skills necessary to solve the problems. The cut-off score was 14 or more of 18 on the original preliminary scoring. In the final data analysis, the total possible score was 21.

The post evaluation, like the individual assessment, was administered in a one-to-one setting of student and investigator. Locations away from the class and the teacher were usually available for the interview sessions, but were not always ideal.

Unlike the individual assessments where the interviewer recorded student responses, students were given a response sheet on which to write their answers and do their work.

They were asked four questions. The first three questions they were asked to respond to initially in the abstract, then a second time after seeing a concrete model of the question. They were then allowed to write a second answer if they wished to change their responses.

For the first question they were shown a 2-square and reminded what a 6-square looked like. They were then asked, "How many 2-squares in a 6-square?" After they responded, they were shown a 6-square and asked if they wanted to change their answer.

At the beginning of the second question they were shown a 2-cube and reminded of a 6-cube. They were then asked, "How many 2-cubes in a 6-cube?" After their response, they were shown a 6-cube which was taped together to hold its shape.

The third question dealt with the growth relationships in reverse. The students were shown a standard box for popcorn and told about a money raising scheme where the popcorn would be sold for 40¢. However, some of the kindergarteners didn't want so much popcorn so we were developing a box which was half the size in each dimension. The students were asked, "What is a fair price for the small box of popcorn?" After their initial response, they were shown a reduced size popcorn box to compare with the standard box and allowed to guess again at a fair price.

The final question was, "If a St. Barnard dog is really a large mouse and is 12 times as tall as a mouse, how many mouse coats are needed to make a coat for a St. Barnard and how many mice are needed to balance a St. Barnard dog on the scales?"

Content Analysis

Test items were written so as to measure student comprehension of the content contained in the Mouse and Elephant Unit. The content consists of concepts, skills, and relationships as described previously. After reviewing the techniques used to measure them, an item by task by content classification will be given for each instrument.

The measuring concepts include area, perimeter, surface area, and volume. Related ideas which also were given attention were rectangle and edges. Since the four main concepts had been given concrete interpretations, concepts were tested in two ways. Either the students were asked to measure a given object or to build a model with a stated measure.

Skills were viewed as capabilities students had for finding measures of area, perimeter, surface area, and volume. The two levels of skills used in reporting student performance were counting and rules. Counting, the lowest skill level, required only that students be able to apply the concrete interpretations of the concepts when physical objects were present.

Rules, however, include any other techniques which required the use of some generalized procedure that shortened the counting process. Thus, organized counting procedures qualified as rules because students were able to shorten the counting process. For example, recognizing that opposite sides of a rectangle are equal would eliminate the need to count both members of the pair.

Rules also include techniques that do not rely on the counting of units. Knowing to multiply dimensions of a rectangle to obtain the area is a good example. The results section will include a listing of specific rules students were observed using. For monitoring and recording students use of measuring skills, there had been originally three categories. Organized counting was considered separate from rules. However, the distinction was often unclear and the two categories were combined.

Applications of skills are also describable in terms of the representational mode of a problem. Thus, students might be confronted with a question in which concrete materials or models are available; a picture is displayed; or abstract descriptions are written or verbalized. Items designated as measuring skills are also identified as being presented in one of these modes.

Relationships deal with variations on one measure when another related measure has been held constant, or with the effects of growth. Area and perimeter as related measures of rectangles, and surface area and volume as related measures of solid blocks, were alternately fixed and varied as students pursued the challenge of four of the activities.

The goal was for students to associate which shapes would result in the maximum or minimum numbers on the varying measure, and to be able to apply this knowledge to the solution of specific problems. Thus, students' understanding of the relationship of variation was sought by specifying one measure, and after students had built or seen a model, ask for different shapes with extreme values on the variable measure. If students had already built an example of a maximum or a minimum value on the variable measure, they could simply respond by saying so.

Growth relationships were the long range goal of the unit. Both the thematic challenges about the Mouse and Elephant and the last two activities were designed to exhibit and explore the effects of linear growth on measures of area and volume. Since the post evaluation focused only on growth relationships, the general description of this test in the previous section told about the techniques used to assess student understanding of this relationship.

The following content analyses; one for each evaluation (a) identify the item number from the instrument, (b) describe the task a student is asked to perform, and (c) label the content which the task is intended to evaluate. In the results section the percent of students successfully completing each task will be given according to the content being evaluated.

To explain how the evaluations were designed to test student comprehension of the content an ITEM by CONTENT by TASK description is given for each instrument.

The Item identification code gives the abbreviation for the test followed by the number of the item on the student test or teacher recording sheet [see Appendix D for instruments]. Since each item asked students more than one question measures of different content are associated with the same item.

The content being evaluated is listed in the second column, followed by a description of the task students were to perform. Whenever the nature of the content or task changed, a separate listing was included.

For the Final Evaluation there is a fourth column containing the number of possible points given to the problem. The letters C and A refer to the nature of the task. C means it was to be executed concretely and A abstractly from a picture or verbal description. Altogether there were 21 total points on the Final Evaluation with 14 of them using concrete materials and 7 from abstract items.

Rectangle Assessment (RA)

Item	Content	Evaluation Task
		Given 3 x 5 rectangle and tiles:
RA 1	Concepts: Area, Perimeter	- measure
	(concept-rectangle)	- identify
RA 2	Skills: Area, Perimeter	Given 7 x 11 rectangle cut out of square paper:
		- describe method for measuring
RA 3	Concept: Area	Given tiles:
	Skill: Perimeter	- build rectangle A = 16
	Relationship: fixed A varied P	- measure
		- build larger/smaller
	Skill: Perimeter	- measure
RA 4	Concept: Perimeter	Given tiles:
	Skill: Area	- build rectangle P = 14
	Relationship: fixed P varied A	- build larger/smaller

Solid Block Assessment (SA)

Item	Content	Evaluation Task
SA 1	(concept and solid block/ edges)	Given cubes: - build 3 by 2 by 3
SA 2	Concepts: Surface Area, Volume	Given 3 by 2 by 3 - measure
SA 3	Skills: Surface Area, Volume	Given picture 3-cube: - measure
SA 4	Relationship: fixed V varied SA	Given pictures: - select arrangement max/min SA, if any
SA 5	Relationship: fixed SA varied U	Given cubes and SA - build maximum U l

Final Evaluation

Item	Content	Evaluation Task	Points
		Given rectangles and tiles:	
FE 1	Concepts: Area, Perimeter(edges)	- measure	3 C
		- label	1 C
	(concept: rectangle)		
		Given tiles and P = 20:	
FE 2	Concept: Perimeter	- build	1 C
	Skill: Area	- measure	2 C
	Relationship: fixed P varied A	- build larger/smaller	2 C
		Given cubes and V = 12:	
FE 3	Concept: Volume	- build	1 C
	Skill: Surface Area	- measure	2 C
	Relationship: fixed V varied SA	- build max/min	2 C
		Given dimensions rectangle:	
FE 4	Skill: Area, Perimeter	- measure	2 A
		Given picture solid blocks:	
FE 5	Skill: Surface Area, Volume (edges)	- measure	3 A
		Given dimensions cubes:	
FE 6	Skill: Surface Area, Volume	- measure	2 A

21 points

(14C - 7A)

Post Evaluation

Item	Content	Evaluation Task
PE 1	Growth Relationship: Area	Find number 2-squares in 6-square: - abstract description - concrete models
PE 2	Growth Relationship: Volume	Find number 2-cubes in 6-cube: - abstract description - concrete models
PE 3	Growth Relationship: Volume	Find cost small box given larger box cost: - abstract description - concrete models
PE 4	Growth Relationship: Volume Surface Area	Given abstract description: - find # mice coats in dog coat - find # mice to balance dog

II. TEACHER DATA

Data provided by teachers of the project was either a description of their teaching experiences or their judgments of students performance. Teaching performance was collected on a daily basis during the unit, at the conclusion of the unit, and ten months later. Judgments of student performance were taken before the unit was taught, on a daily basis during the instruction of the unit, and again at the conclusion of the unit. Following are descriptions of the techniques and instruments used to gather the information.

Teaching Performance

Daily Teaching Logs: Each teacher not being videoed was asked to keep a record of what happened as they attempted to implement the script. Of interest were student responses, elaborations of teacher reactions to and modifications of the script.

The information gathered in the daily logs is discussed in the results section and summarized in Appendix C. As would be expected, much more information was available from the four videotaped classes than from the others. Unfortunately, the four video teachers were not asked to complete their logs because of the presence of the observers and their logs. In retrospect, the teachers' reflections would have been interesting.

Teachers being videoed had been exempted from keeping the written daily log in anticipation of obtaining verbal comments during daily interchanges between investigator and teacher in some form of stimulated recall sessions. Early plans called for their viewing tapes of lessons to give recollections and retrospective comments. However, as it turned out there was no time period in which this could be done. With so many focal points in this exploratory study decisions were made to maximize opportunities for hypothesis generation about all aspects of the study rather than to insist on a particular method.

Debriefing Sessions: At the conclusion of the unit each teacher was interviewed by an investigator while the other investigator conducted the final evaluation with the class. In addition to gathering data on students, teachers were asked to make comments about teaching the activities. These comments were incorporated into a general description of teacher reactions.

Follow-Up Questionnaire: Ten months after the unit was taught the teacher was asked to respond to three specific questions regarding the unit. Those questions were:

Question 1. In retrospect, what did you think of the Mouse and Elephant Unit?

Question 2. Did teaching the unit require you to teach math differently than you usually teach it?

Question 3. Did your participation in the unit affect the way you now teach math?

Teacher Judgment of Student Performance

Pupil Sorts: Using cards with a student's name on each card, the teachers were asked to sort their students prior to the teaching of the unit and again after they completed the unit. On the initial sort they were asked to judge their students on the basis of mathematics performance, mathematics potential, and involvement in class. Only one of these was repeated at the conclusion of the unit as teachers were asked to sort their students according to performance during the unit. Teachers were also asked to predict students' performance on the Final Evaluation by dividing them into four ordered groups.

Item Analysis: Predictions of test performance were also made on an item-by-item basis in a manner chosen by the teacher. Thus, some predicted how many students would respond correctly while others only used general descriptions.

Daily Report on Individual Students: Teachers were asked to make daily judgments as to whether students had acquired the content germain to the activity of the day (H), sort of acquired the content (S), or did not have it (D). A fourth category enabled teachers to respond if they had not been able to form a judgment (N).

Teachers were also asked for further details on student performance. They were asked to indicate the skill level at which students were functioning. This data was, for all practical purposes, not provided. Teachers were encouraged to comment on difficulties or insights they noted and if extra challenges were offered to a student.

Observer Data: The observer roles were assumed by the co-investigators of this exploratory study. Observations were confined at least with regard to data reporting, to the four videoed classroom. Every teacher's classroom was visited at least once, and almost always twice. The purpose was to gain some comparative feeling for the experiences of different classes and teachers, and give the students an opportunity to become accustomed to the presence of the investigators before they administered the evaluations.

Records of what went on in the four videoed classes are contained in the tapes and classroom observation forms.

Video Records: Video records were maintained on four classes. During instruction of the unit both investigators were usually present. One was confined to the camera while the other was able to move about the room more freely whenever the lesson allowed. The investigators exchanged roles from class to class so each was the observer in two classes.

Every day the unit activities were being taught was videoed. However, taping was not continuous over the entire class period due to limited sources of tapes. The camera was run continuously during the early classes in the project. Later, the camera was left on during the portions of the activities in which the teacher was conducting the lesson. Most of the "down times" were chosen to coincide with the work time called the exploratory phase.

Taped records allowed for repeated and more intense examination of the instruction. This approach, similar to micro-ethnography, results in the generation of hypotheses which coincides with a primary function of exploratory research.

It had been anticipated that video tapes would also be used in stimulated recall sessions with teachers or students', but that was not the case. The decision to eliminate this technique was based on the difficulty in finding time and location condusive to the task; also the questionable value of the data obtained for the sacrifices required; and a willingness to err in favor of monitoring the diversity of four teachers rather than producing more in-depth descriptions of fewer teachers. This was, after all, a small grant, exploratory study.

Classroom Observation Forms: There were four different observer forms used each day a class was being videoed.

(1) A Classroom Interaction form (yellow) was used by the observer to record a general running description of the progression of the activity, particularly during the phases of instruction when the teacher was teaching the whole class.

(2) A different form (white) was used during the exploration phase, the work time. Students names were organized by groups seated around tables to make the observation and data collection more efficient.

(3) A Daily Record form was completed by the person on the camera to provide an index to the video tapes according to the instructional sequence of events.

(4) A log of Recording Information (white) provided such things as day, activity, time, tapes, student absences, and which students were given individual assessments. This data was later transcribed to a Teacher Log (yellow) compiled while viewing the video tapes.

RESULTS

In this section the results of the data gathered regarding student learning and affective responses, teacher judgment and reaction to the unit and various observations by the investigators will be presented.

STUDENT DATA

The data pertaining to students concerns the learning of concepts, skills and relationships, both variability and growth. Results of items from the Individual Assessments of Rectangles (RA) and Solids (SA), the Final Evaluation (FE) and the Post Evaluation (PE) are reported according to the content being measured.

The percent of students giving the correct response to a question on task is listed in a table for the appropriate content along with the identifying label for the test and test item number. A listing of test items by task and content can be found in the previous section on evaluation procedures for student learning.

The Individual Assessments on Rectangles and Solids which were administered only to students from the four video-taped classrooms were taken by 106 students (97%) and 80 students (73%) respectively.

Students from all classes were tested with the Final Evaluation. A total of 350 students actually took the written test, 93% of the students identified as members in the thirteen classes. Of the 27 students not tested about 20 had been absent between 4 and 8 days during instruction of the Mouse and Elephant Unit.

The Post Evaluation was only administered to 38% of the students who took the Final Evaluations. These were the students who scored at or above the selection cut-off score.

Observational comments are based on student performance in the activities made by the investigators and teachers. Student reactions to the unit as a whole were obtained from students taking the Final Evaluation. Twelve of the thirteen classes were asked their reactions so a maximum of 325 were given the opportunity to write their comments; a few did not respond.

The Final Evaluation was the only test for which scores were obtained; all others were reported only by items as was the Final Evaluation.

Out of a 21 points possible score on the Final Evaluation, students averaged 12.9 points with a standard deviation of 4.7. This total score was also broken into subscores according to whether the item had been based on using concrete manipulatives or had been worked at a more abstract level then pictorial or written. The mean concrete score, with a possible 14 total, was 9.69 (s.d. 3.2) and the mean abstract score was 3.2 (s.d. 1.95) out of 7 possible. But these scores are not as meaningful as the item-by-item descriptions reported according to content.

A. Concepts

The basic measuring concepts of area, perimeter, surface area, and volume are described in terms of concrete materials. By simply knowing what and how to count students could find the measures from physical models. To evaluate the student understanding of these four concepts, problems asked students either to measure a concrete model or to build a concrete model with a specified measure. At least one such item for each concept was included on both the final evaluation and the individual assessments.

Student Success Rates: Overall there was about an 80% success rate on the eleven items measuring student understanding of the four basic concepts (see Table 2). Success rates of items measuring the same content were quite consistent with one another on both tests and on both measuring and building tasks. However, comprehension was not consistent across concepts.

TABLE 2

Percent of Students Demonstrating Understanding of Concepts on the Final Evaluation and Individual Assessments

	Concept			
Task	Area	Perimeter	Surface Area	Volume
measure a concrete model	94.6 (FE 1)* 96 (RA 1)	80.6 (FE 1 75.5 (RA 1)	58.3 (FE 3) 64.6 (SA 2)	89.7 (SA 2)
build model to specified measure	94.3 (RA 3)	75.7 (FE 2) 88.0 (RA 4)		83.7 (FE 3)

*Test and Item Identification

Area was the best understood concept with about 95% of the students correctly measuring or building a rectangle of a specified area. Volume was next with almost 90% of the students individually assessed giving the correct measure and 84% building an acceptable solid block on the final evaluation.

Perimeter came out third with success rates varying from 75.5 to 88%. The most difficult concept was surface area. Roughly 60% of the students answered the two items correctly.

Difficulties: The fact that students learned the concepts of area and volume better than perimeter and surface area is more easily understood when the units being counted are considered. Measures of area and volume are found by counting the very objects used to build the models. A rectangle formed with 12 tiles has an area of 12, and a solid block formed with 12 cubes has a volume of 12. But to measure the perimeter or the surface area the edges or the square faces had first to be isolated from the tiles or cubes before they could be counted. This required an additional discrimination task and was the probable cause for greater difficulty in learning these two concepts.

The 80.8% success rate in measuring perimeter on the Final Evaluation (FE 1) can be compared to students success in measuring the edges of the same rectangle. It turned out that 95% of the students gave the correct measures for edges. Thus, the nearly 20% errors in finding this perimeter were not caused by students being unable to measure the edges. Similarly, on the item (FE 5) students were able to find the edges of a solid block when shown a picture. Again suggesting that students could distinguish the edges.

This difference in comprehension of the concepts was also related to an interesting and reasonable, though unanticipated, association. Apparently students found the concrete interpretations to be stronger organizers then the theoretical constructs. Students related concepts of area and volume as both measuring the inside objects, which was the same attribute which made these concepts easier to learn. Similarly, they related perimeter and surface area as being the measures 'around'. These associations were in contrast to the mathematical linking of concepts which share the same unit, such as area and surface area.

Introducing perimeter as the number of segments around the rectangle did not overcome the misconceptions students generated in attempting to learn the concept. Instead of counting the string segments or tile edges, some students were observed counting the outer ring of tiles still within the boundary of the rectangle, others lined up tiles around the outside of the boundary, and others counted knots instead of segments.

Unless the actual measuring process could be observed during the test, the only way of determining which of these might be used would be a detectable error pattern. From the observations made during various evaluations and classroom observations, including the Final Evaluation, it was apparent that students went about measuring existing rectangles drawn on paper in one of three ways. Using the tiles and (a) covering the rectangle completely, (b) arranging them around, either within or outside the boarder of the rectangle or (c) laying the string along each edge or around the entire rectangle.

Cubes, on the other hand, were simply used to build the solid blocks and there were no variations in this procedure.

Embedding concepts in stories did more than initially convey their meanings; it provided a link between their natural language and the less familiar language of mathematics. Forgetting or confusing which words signaled a concept underscores the distinction between knowing the meaning of a concept and knowing the proper mathematical word to express it. Students could more readily recall the rectangle word to identify the shape when asked during the Rectangle Assessment (.97%) than the Final Evaluation when the numbers dropped to

72.6%. Recall interference was evident in that some students wrote "triangle" while others said "square" for the 4 x 5 rectangle or simply left it blank (a possible perceptual rather than measure error).

Although surface area was the most difficult concept according to the test results, students were not observed applying any misconceptions other than failing to realize that the squares had to be counted on the bottom face even though it was not visable. The larger number of errors associated with measuring surface area (close to 30%) were due to misconceptions, or careless counting.

B. Relationships

There were two general categories of relationships explored during the Mouse and Elephant Unit: variability and growth relationships. Because of the differences in the categories and how they were treated, descriptions, data, and discussions of the relationships will be reported separately.

Variability Relationships: Variability relationships refer to the variations on one measure when another related measure is fixed and the generalizations as to which shapes would produce the maximum and minimum variations. Fixing the perimeter of a rectangle, for example, still allows for variation in the shape and the area of the rectangle. The most square-like shape produces the greatest area while the longest rectangle results in the smallest area.

There were four variations with rectangles and solid blocks included in this category of relationships. Each relationship was the basis of exploration for a different activity. There were 12 items for examining student understanding of the four variability relationships, of which only four items examining only two of the relationships were included on the final evaluation. Essentially, students were given a fixed measure and asked to build or measure one concrete example, which was the portion of the task labeled as measuring concept understanding. Next, students were asked to produce and report alternative examples on the variable measure which were larger/maximum or smaller/minimum.

Success Rates: The questions on which students had to demonstrate concretely their understanding of the variability relationships were successfully completed by about 60% of the students (see Table 3). Only one item asked only for recognition of the relationship from a picture and it was spurriously high in comparison with an 87.5% success rate. Although the results across all items about the four relationships were within a range of 25% of one another, there are two patterns warranting attention.

TABLE 3

Percent of Success on Variability Relationships

Variation of interest	Fixed Measure Area	Perimeter	Surface Area	Volume
larger/ maximum	71.7 (RA 3)	50.3 (FE 2) 54.7 (RA 4)	46.2 (SA 5)	61.4 (FE 3) 73.0 (SA 4)* 87.5 (SA 4)*
smaller/ minimum	71.7 (SA 3)	53.1 (FE 2) 44.3 (RA 4)		63.7 (FE 3) 75.0 (SA 4)*

(Test and Item identification)

*Pictorial Presentation

First, the four relationships were not demonstrated with equal success. The differences across the four relationships, though not large, reflect the same difference that occurred across concepts.

Students were better able to find extreme measures when the fixed measure established the number of tiles or cubes just as they were better able to measure with tiles and cubes serving as the unit.

Secondly, within each relationship there was a definite similarity in success rates across the two variations sought. Students did about as well finding the shapes with the larger/maximum measures as with the smaller/minimum measures.

Difficulties: When the area or volume was fixed, so was the number of tiles or cubes. The students needed only to concentrate on rearranging them and measuring the desired variation to find the best solutions. However, the reverse tasks of fixing perimeter or surface area required manipulating three variables. Students not only had to work with different arrangements of tiles or cubes; they also had to vary the number as well to obtain measures of the variations sought. From a cognitive processing view the latter task had more potential for error as the results indicated.

When working with concrete objects students were able to explore and demonstrate their understanding of variability relationships with a success level roughly 20% below that attributed to understanding concepts. This is significant because these relationships are often ignored in classrooms and because without the concrete embodiment the approach would tend to be highly abstract. Applications of skills demonstrated on the evaluations would indicate that the pedagogy of an abstract approach should at least be questioned.

The similarities across the two extremes within each relationship suggests that if a variability relationship was understood, it was understood in its entirety. That is, if the generalization or ability to find one extreme was demonstrated, there was a greater likelihood that the other would be found as well. However, data taken from the final evaluation sheds some doubt on the validity of this conjecture.

There was some slippage as the number of students finding both extremes was not the same as those finding each extreme separately. For the rectangle problem, 37% of the students actually found a rectangle with both a larger and smaller area than the one they originally built. This is lower than the near 50% success reported for each extreme. Similarly, the success rate for finding one solid block of 12 cubes with a maximum or minimum surface area was in the low sixties while only 48% could find both. This suggests that the initial rectangle or solid block built to exemplify the fixed perimeter or volume may have had an impact on the subsequent success in finding the extremes. Since the data was available, this idea was explored (see Table 4).

TABLE 4

Responses of Students on Final Evaluation
When Building Rectangles With Perimeter of 20

Initial Response	% of Students	Conditional Probabilities of finding: larger area	smaller area
1 x 9	13.6	86.1	86.1
2 x 8	21.1	68.7	65.1
3 x 7	12.1	74.8	68.5
4 x 6	30.9	40.2	59.7
5 x 5	22.3	52.5	57.6

When asked to build a rectangle with a perimeter of 20, most students chose a 4 x 6. Thirty-six percent responded with a rectangle having either a maximum or minimum area while only 12% formed one with medium area.

In terms of the conditional probabilities for finding the extremes, given the initial choice, the chances are somewhat varied. The initial choice with the greatest chance of demonstrating understanding of the variability relationship was a 1 x 9 while the least chance of success was associated with a 4 x 6. This is reasonable considering that with a 1 x 9 the students could hardly fail to find a rectangle with a larger area or to recognize that an edge of one was the smallest possible to form with tiles. On the other hand, starting with a 4 x 6, the students had used all the tiles which were usually available to them in their regular classroom. It was possible that a student never considered that a rectangle could be built which satisfied the perimeter requirement and used more than 24 tiles. Also some of the students may still have been rejecting a square as a rectangle. It is surprising that not more than 60% could find a different rectangle with smaller area since anything else they could build would satisfy.

Another surprising result was that students who offered a 5 x 5 initially were only successful 52% of the time in saying there was not another rectangle with a larger area. Perhaps having to declare that fact, at least for the extreme, was more difficult for students not oriented to such questions. But again, there was no more success in finding one with a smaller area. Evidently some students did not comprehend the variability relationships or the problem.

TABLE 5

Responses of Students on Final Evaluation When Building Solid Blocks With Volume of 12

Initial Response	% of Students	Likelihood of finding: Maximum	Likelihood of finding: Minimum
1 x 1 x 12	6.8	55.0	50.0
1 x 2 x 6	14.7	72.1	65.2
1 x 3 x 4	21.5	65.0	61.9
2 x 2 x 3	57.0	69.5	80.3

Similar data was obtained on the problem in which students were given 12 cubes to package, asked to build an initial example and then find which packages would have the maximum and minimum space armor costs (surface areas). The most popular initial response given by 57% of the students was to build the most compact package with the minimum surface area (see Table 5). It was also the most advantageous initial response for finding the minimum cost of wrapping 12 cubes. Although the success rates for the extremes were slightly less when students started with a 1 x 1 x 12, this was probably attributable to the fact that so few did (6.8%).

Growth Relationships: The second type of relationship, the growth relationship, was the focus of the thematic challenge about the Mouse and the Elephant and the last two activities of the unit. A growth in linear dimensions of squares or cubes results in growth in area, perimeter, surface area, and volume. The generalization sought was that a growth in dimensions from 1 to n would increase area by a factor of n^2 and volume by a factor of n^3.

Because the growth relationships were highly dependent on having already acquired both concepts and measuring skills beyond counting, only students who did reasonably well on their final evaluation were tested individually on a post final evaluation. Thus, data on growth relationships comes from 134 students which is the top 38% of the students in the project (see Table 6).

TABLE 6

Percent of Students Successful on Post Evaluation

	(1)* Squares	(2) Cubes	(3) Popcorn	(4) Mouse/Dog Balance	(4) Mouse/Dog Coats
First Response (abstract)	42.5	16.4	15.7	11.9	7.5
2nd Response (concrete)	49.3	30.6	18.7		
Total	91.8	47.0	34.3	11.9	7.5
Estimated % of Entire Population	35	18	13	5	3

*(item number)

Success Rates: The only growth relationship which students were able to demonstrate with much success was the two dimensional square question. Ninty-two percent of the students were able to figure the number of 2-squares in a 6-square and over half of those required a look at the model of the 6-square to do it. The success rate was only about half as good for the cube problem as for the square problem. Forty-seven percent found the number of 2-cubes in a 6-cube but only a third of those (16%) could do so in the abstract mode.

A shrinking rather than growing box of popcorn was correctly answered by even fewer students. Only a third of all the students tested figured out the fair price, but the number doing so without seeing the smaller box was the roughly same as for the cube question when the larger cube had not yet been shown.

Transfer from the solution of the Mouse and Elephant questions to the Mouse and Dog questions on the test was almost nonexistant. Twelve percent found the number of mice needed to balance the elephant and 7.5% found the number of mice coats needed to make a coat for a dog.

However, all of these results are based on data from students already scoring in the top 38% on the final evaluation. To put the above figures in perspective requires estimating the number of percent who would have answered correctly out of the entire student population. This was done by multiplying the total percents in Table 6 by 38%.

Difficulties: The tasks presented in this post evaluation were, except for the square problem, simply beyond the cognitive skills of the students. The fact that students were unable to visualize the described objects with much accuracy was most apparent when they registered surprise at the immensity of the 6-cube or the tinyness of the small box of popcorn.

It was quite apparent, too, that students attempted to handle the first three problems in a concrete manner even when they had not seen the second model. There were movements of the hands and eyes that revealed they were attempting to count from their image of the missing object. Also, students attempted to partition the larger object into the smaller ones. No more than five students actually computed areas or volumes and divided to find their answers to the square and cube problems with an entirely abstract approach.

Although the students were told and it was physically indicated by pointing to the original box of popcorn during the description, students ignored the halving of all three dimensions and halved only one or two instead.

There were two frustrating experiences for the students taking this test. Many found the cube problem too difficult to handle and their conclusion was obvious from their actions and comments. However, they tried and many, some mistakenly, felt quite comfortable with their responses. The Mouse and Dog problem was truly too challenging. Many students did not respond at all while others admittedly wrote down guesses. Still some others took it as a personal challenge and did not want to give up without the answers which they simply could not find. These students sometimes had to be forced to quit.

C. SKILLS

To have a measuring skill necessitates both having the means to find a measure and the ability to apply it. The extent to which students had acquired measuring skills is reported in terms of the specific skills they used and the representational mode of the problems to which the skills could be applied.

Nature of Skills Observed: There are essentially two types of measuring skills: counting and rules. Counting is a basic skill in measuring concrete objects for it requires only that a student know what to count, the concept, and be able to count it. Assuming that the students had learned the concepts, as the test data suggests, they were equipped with this basic measuring skill.

Measuring skills designated as rule-governed procedures ranged from organized counting to calculations with rules. Organized counting was classified as a rule because it implied the use of a generalization other than the conceptual interpretation. For example, rather than counting the squares on each face of a solid block students would capitalize on the fact that the opposite faces were the same size.

Detailed records as to the particular skills individuals used during the tests and activities were, unfortunately, not possible to obtain. There were two descriptions of the nature of skills students did use, however. One is a list of the rules students were observed using on the individual assessments as well as during the unit (see Table 7). The other is a quantified record of the type of skill being used on several items of the individual assessment.

While the list of students' rules may be incomplete it does reflect the most frequently observed routines as well as some attributed to a few. What is missing is the relative frequency with which each occurred. In expressing these rules an effort has been made to preserve the essence of what students verbalized. What students were not observed doing was to write a rule, substitute in the appropriate measures and perform the calculations which is the typical textbook demonstration. Instead, students were executing a routine in a step-by-step manner usually in their heads. When the arithmetic became too much to retain, submeasures were written down so that the operations, addition for the most part, could be carried out.

TABLE 7

Rules Students Used for Measuring

Area
of a rectangle ---- (bottom edge) times (side edge) = A
(where the numbers were obtained by counting tiles or segments.)

Perimeter
of a rectangle ---- b.e. + b.e. + s.e. + s.e. = P
(b.c. x 2) + (s.e. x 2) = P
(b.e. + s.e) x 2 = P

of a square ---- edge x 4 = P

Surface Area
of a solid block -- (area of top face x 2) + (area of front face x 2) + (area of side face x 2) = S.A.
(where the numbers were obtained by counting squares or using area rule.)

of a cube ---- (area of one face) x 6 = S.A.

Volume
of a solid block -- (area of one layer) x (number of layers) = V
b.e. x s.e. x ht. = V

It is noteworthy that the procedures as verbalized by the students often did not conform to the more common ways the rules are likely to be expressed by teachers, or in textbooks. Perimeter, for example, is usually expressed as 2 x (b.e. + s.e.) rather than either of the two listed above.

As can be seen from Table 8, the primary measuring skill demonstrated during the individual assessment was counting. This was true regardless of whether the problem was presented with concrete models or pictures. Of course, a direct application of a concept would most likely be done by counting, unless the numbers were rather large. Except for the picture rectangle, the dimensions were relatively small in all problems.

TABLE 8

Percent of Students Using Counting or Rules on
Concrete Models (I) or Pictures (II)
During Individual Assessment

	Measuring							
	Area		Perimeter		Surface Area		Volume	
	I	II	I	II	I	II	I	II
counting	78	88	57	66	58	72	74	70
rules	22	12	43	34	42	28	26	30

The percent of students displaying rules was never large, but it was greater for perimeter and surface area, which are the 'around' concepts. When the opposite sides of rectangles or opposite faces of solid blocks were recognized as being the same, students were more likely to incorporate this duplicity into an organized counting procedure and thereby avoid duplication in counting.

A slight decline in the number of students using rules can be observed when the switch was made in presentation of the problem from concrete model to picture for each type of measure except volume.

Applications Across Representational Variations: The second way in which student understanding of skills was examined was through applications of skills to problems with varying modes of representation. Problems were selected in which the rectangles or solid blocks to be measured were displayed concretely, pictorially, or by written description giving dimensions.

Testing with problems according to the mode of representation provides information as to the nature of the skills being applied. The argument is that the more abstract the problems become, the more likely abstract rules are needed for finding the solutions. Counting as a skill is associated with concrete models, whether real or imagined.

Items already labeled as measuring concepts were not included as measuring skills even though they qualified as concrete representations in which a skill was required. Of the 12 items evaluating students' abilities for applying their measuring skills, there were no items for measuring volume with concrete objects or for measuring rectangles presented pictorially.

Success Rates: Students demonstrated nearly equivalent skills for measuring area, perimeter and surface area with concrete materials. Close to 80% of the students were able to give the correct measures for area and perimeter while 70% could correctly measure surface area (see Table 9). Again, these figures are somewhat less than the percents associated with understanding the concepts of area and perimeter. But unlike the unencumbered requests for measures of objects on concept items, the skill questions were embedded in more complex tasks which, if not completed would result in no response. Surface area was an exception in that more students were credited with finding surface area from the skills items than from the concept items.

TABLE 9

Percent of Students Demonstrating Measurement Skills

Problem Representation	Area	Perimeter	Surface Area	Volume
Concrete	86.3 (FE 2) 79.4 (RA 4)	78.1 (RA 3)	71.4 (FE 3)	
Picture			48.9 (FE 5) 57.5 (SA 3)	43.1 (FE 5) 53.7 (SA 3)
Abstract written description	56.3 (FE 4)	44.0 (FE 4)	17.1 (FE 6)	24.6 (FE 6

(Test and Item Identification)

As the representation mode of the problems became more abstract the success rates for measuring rectangles and solid blocks diverged rather dramatically. Success in measuring surface area and volume from the picture of a solid block approximated what students could do given only the dimensions of a rectangle in the abstract representation. However, the success level was down in the neighborhood of 50%.

Students were far less able to compute the surface area and volume of an 8-cube when the abstract or written description was the only information available. Only about 20% of the students were successful in finding these measures.

Discussion About Skills: Students did not acquire skills adequately to solve problems across varying modes of representation. Counting, the sole skill for as many as half the students, was inadequate for measuring without physical models. There was no evidence that these students possessed the rules to compensate for what could not easily be visualized and counted. From the positive incidents of rule application it was apparent that rules governing counting; organized counting procedures, were learned more readily than were formalized rules necessary for handling the most abstract problems.

The extent to which dependence on concrete representation, effort, or previous exposure to the rules accounted for the test results is impossible to separate out. However, given the many opportunities students had to measure with physical models over the eight or more days of the unit, the small amount of transfer to abstract problems suggests several possible explanations. For one, the varied levels of cognitive development known to exist in students of this age may account for some of the variation in acquiring rules. A lack of formal operational thought processes could inhibit what was learned or the ability to translate it to other situations. Also, even two weeks may not be enough time to acquire all the content at the mastery level and certainly not without some practice in applying the rules that emerged. Furthermore, the degree to which students were actually exposed to various rules was dependent upon the individuals, peers, and the teachers. Intensive observations of four videoed teachers revealed there was substantial variation in what teachers did to enhance the rules. But rules were not necessary for students to participate in the interesting challenges of the activities.

The 10% rise in success measuring surface area from the concept items to the skill items may well be explained in terms of what the rates signified. Each concept item asked only that one measure or object be found. However, on the skill items there were two opportunities to measure solid blocks being formed in response to a more complex problem. That is, 71% of the students were able to give at least one of those two measures correctly. Overall task interference would probably account for the fact that only 45% were able to give both measures correctly. What this suggests is than an earlier conjecture about student difficulty in measuring surface area being due to failure to include each square on every face may well be true. Having to be correct only once out of two times would be more likely under such conditions as long as the concept was understood.

The lack of agreement in success rates for measuring rectangles and solid blocks given only the dimensions suggests the difficult was inherent in the task of measuring in the third dimension. Imaging an object which, when present, is entirely visable must be easier than imaging an object which, even when present, has hidden portions. Rectangles would be more easily visualized or even drawn knowing only the dimensions, than cubes. In fact, a number of students did draw pictures to accompany their abstract rectangle problem while there was no such evidence for the cube problem.

D. STUDENT PERFORMANCE IN THE ACTIVITIES

From the systematic classroom observation of the four classes, further study of the video tapes, and occasional visits to the nine other classes, we are able to make some general comments about the manner in which the students encountered the concepts and relationships, the variety of skill levels which appeared, and the difficulties and misconceptions which were observed.

Activity 1. Many students had encountered the concept of area previously, frequently knowing it as length times width. The idea of counting squares was usually new. The name and concept of perimeter was generally not known. The script was careful to use the words "bottom edge" and "side edge" to describe a rectangle because of the difficulties the words "length" and "width" presented during the pilot classes.

Many difficulties were observed in measuring perimeter. The students were provided with marked strings calibrated to the tile edge if the string was held taut correctly. They frequently counted the marks rather than the segments. They also tended to count the border tiles rather than the border segments. The banquet tables and the seated persons seemed to provide the clearest portrayal of perimeter.

The first activity (and activity 4) are different from the other activities in that they contain no challenges which deal with relationships. They only introduce new concepts then give the students practice in measuring the new concepts.

Activity 2. The students first encountered the variability relationship when they were asked to fix the area at 24 tiles and find the variety of perimeters. This task was reasonably easy for most of the students as the number of tiles remained constant and only their rearrangement was necessary. A quick review of the meaning of area and perimeter seemed sufficient for most students. The idea of cutting a paper model of square cm. paper to represent a rectangle consisting of square inch tiles was introduced and immediately understood by nearly all students. There seemed to be a general appreciation of the pattern by the students in those classes where the teacher displayed the cut-out models on the overboard overlaying the 1 x 24, 2 x 12, 3 x 8,...,24 x 1 and observing the hyperbolic form. There also seemed to be a general understanding of the rectangles which would provide the maximum and the minimum perimeter.

Activity 3. It became clear very quickly that the challenge in this activity was significantly more difficult than the previous activity. Now the study of the variability relationship requires the perimeter to be fixed at 24 and the variety of areas is recorded. This challenge required the students to add or remove tiles rather than just rearrange them. The complete solution of the challenge also required the students to pool their tiles in order to form all possible rectangles from 1 x 11 to 6 x 6. The students generally seemed to understand which shapes provided the minimum area and which provided the maximum area.

When the models were layed on the overhead ordered by the bottom edge, some students expressed disappointment in the linear stair steps pattern which developed as compared to the hyperbolic pattern in Activity 2. However, those students in classes where they had an opportunity to follow with a graph of the bottom edge against the area generally were very interested.

Activity 4. The general consensus was that Activity 4 was in some ways the most interesting both for students and teachers. The students are introduced to the concept of surface area through the challenge of cutting from square cm. paper a "space armor jacket" to protect a concentrated cube of space food. The cube is a 2 cm. cube from the set of geoblocks. The cost of the jacket is one dollar per square cm.

This activity is highly dependent on some spacial abilities and the variety of skill levels becomes obvious very quickly. Some students can cut out a proper jacket at the first attempt while others are frustratingly trying to wrap the cube like a present, not knowing what to do with the overlap. In several classes certain students would become intrigued with the challenge of finding as many different shaped jackets were possible for the cube. Once they realized they could subdivide the individual squares it became apparent that the answer was infinite.

As they were able to move to other recangular solids, and then to triangular pieces, there is a sufficient variety to continue the challenge for a wide ability class. The differences in skill level in counting the squares to find the cost of the space jacket was very apparent. Some students found ways to approach it systematically but many students continued to count square by square.

Activity 5. The students were presented with a bag of 24 color cubes each 2 cm. on an edge. Through the experience in the pilot classes we learned to provide a single color to each student as time was inevitably spent exchanging cubes and sorting by color when they were presented with an assortment.

As the students built in three dimensions the new terms of "bottom front edge", "bottom side edge" and "height" were introduced to enable the class and teacher to communicate about the solid blocks they were building. The challenge, as in Activity 2, asked to keep the number of cubes constant, but to rearrange them and determine the varieties of surface area (space armor costs). There was little difficulty for the students to shift the size of a square from a square cm. to the face of a cube which is four square cm. However, there was great difficulty in counting the surface squares accurately for many of the students. Only a few students began using organized counting methods.

Many students seemed to understand that the maximum surface area was obtained by the "stretched out" forms while the minimum surface area was obtained with compact solids.

Activity 6. Just as when moving from Activity 2 to 3, this activity presents a much more difficult challenge as cubes must be added or removed to examine the variations. The concept of volume seems easily understood as the number of cubes and the computation of volume by multiplication was prevalent. The challenge of maximizing volume for a certain restricted surface area was hard and produced a lot of frustration.

In general, these two activities did not seem to provide sufficient experience for many of the students to get these ideas sorted out very well. In our most recent pilot work we have been developing some alterations which may help students better understand the relations between area and volume.

Some students in some of the classes arrived at the conclusion that a cube would provide the maximum volume for a given surface area.

Activity 7. Returning to two dimensions and the tiles, the concepts of area and perimeter are reviewed very easily but the new gorwth relationships are encountered for the first time. The linear increase of perimeter and the quadratic increase in area became clear as the patterns were recoded in the summary. When students had available to them a large variety of paper model cut-outs of various sized squares, they demonstrated a wide spread understanding of the number of m-squares in an n-square when m divides n. However, there was very little evidence of an abstract generalization of these growth relationships.

There was generally no transfer at this time to the related problem of the mouse and elephant coat so the flat surface usually led the students to think of only one side of the elephant, not the covering around it.

Activity 8. In this culminating activity the concepts of volume and surface area are viewed with ease. The growth relationships of surface area and volume in a growing cube are begun to be appreciated.

Classroom management problems always arose potentially when the cube size increased beyond an edge of 3. The students had to be grouped carefully and their cubes pooled in order to obtain the 4,5 and 6 cubes.

This activity was impressive in appreciating how fast the volume of a growing cube increases.

With care in computing the cost of a wrapping for a 1-cube and the larger cube the answer to the surface area question began emerging, from the summary patterns. The volume pattern seemed rather clear also.

The number of students who pursued the challenges for the mouse and elephant questions complete to the solutions was generally very few. The numbers generally displayed, but they were so large, 1600 and 64,000, they seemed to be without much meaning.

E. STUDENT REACTIONS TO THE UNIT

Just before the students began to respond to the final evaluation, they were asked to write on the back of their papers what their impressions were to the mouse and elephant unit. The responses were usually very brief and took a variety of forms. Some students reacted to specific aspects of the unit such as the cubes or the string or the space armor. Others wrote globally in praise or in contempt of the unit. The use of manipulatives seemed to be a very positive factor in the students reactions. The responses were categorized and are shown below in Table 10.

TABLE 10

Student Reactions to the Unit

	Number of Students	Percent
Positive - general/global	146	44.9
Positive - with specifics	82	25.2
Better than regular math	6	1.8
Mixed response (some positive/some negative)	31	9.5
Negative comments	12	3.7
Repugnant - more violent negative	5	1.5
No response	43	13.2
Total	325	

Of the 282 students who responded, 72% felt positive about the experience while just over 5% reacted negative about it.

A major proportion of the negative reaction came from the class of Teacher 2 who had spent a lot of time toward the end of the unit drilling the class on the concepts and skills. The result seemed to be a reaction against the unit which showed on the other final evaluation scores as that class scored lower on the 3-dimensional items then might be expected. The longer the unit lasted in an atmosphere of high pressure, the more the learning and the attitudes deteriorated.

Class 9 was not asked to respond due to an oversight.

A. TEACHERS' PERFORMANCE OF THE INSTRUCTIONAL PHASES

This section describes the nature of variation exhibited by teachers in executing the script for the Mouse and Elephant Unit. But the question is variation from what? At the time the proposal for this study was written the questions centered on deviations from the script and content, at least with regard to accuracy in portraying the content. However, as the study has progressed the focus of interest has shifted.

Difficulties students encountered may have been the result of the particular challenges or concepts within the activity, to deficiencies in the script which failed to give the teachers adequate direction, or to the teachers execution of the script. The more evident effects of teachers' execution of the script are what these results are attempting to address.

Activities for the unit were organized around a task or a challenge and the mathematical ideas associated with it. Thus, a script was an attempt to assist teachers in presenting the challenge, allowing students to work on the task, and then looking for interesting mathematical patterns, relationships and generalizations. From our past research and piloting experiences, a sequence of directions to the teachers was compiled. It was an attempt to provide teachers with sufficient guidance so as to have the activities run smoothly and as envisaged.

During the process of revising and test piloting the script it became apparent that the teaching might better be described in a way that highlighted the different roles that the teachers were to play. The result was the labeling of three distinct phases of teaching the activities as launching, exploring and summarizing.

The sequence of directions in the script was not clearly separated into these three phases but they were inherent in the ordering. Issuing of a challenge was intended to signal the exploration phase. The word summarize was used in the script to signal time for teachers to call the class back together.

When observing the teaching in the classes and viewing the video tapes we were able to record many specific behaviors and techniques teachers used in the particular phases. We could also observe the effects of those behaviors and techniques on the smooth conduct of the activity.

Thus, the generation of our model of instruction has been an evolving process in which the specific roles and techniques of the teacher in executing the particular phases are becoming much more explicit. Many specific behaviors we originally thought to be optional and without serious effects now seem to be more important. Discussion of those specific behaviors and techniques follows in the next sections.

Launching Phase: For the ideal activity, the launching phase consists of three stages: (1) presenting new concepts and reviewing old, (2) conducting a mini-challenge, and (3) posing the major challenge.

Presenting new concepts or reviewing old concepts and skills is essentially accomplished by example and story embellishment.

Students are first asked to build or display a model from which they can obtain measures on the concepts by counting the number of units. Each student is to have access to the materials and to perform the task. This initial request is usually done through a story in which the interpretation of the concept provides an alternative language and also relates to an action.

When asking the students to build something with concrete materials teachers can easily offer feedback to students by simply looking around to see what is built. This feedback is best done in a rapid-fire, yes/no fashion. As soon as several of the students have completed the task, a correct example should be displayed or a measure should be shared. This is how students check to see if they are correct or, if wrong, are motivated to try again. Consequently, the pace of this portion of the lesson should be fairly fast.

Usually this first example is followed by several more. The number of examples is left to the teacher's discretion so as to accommodate the needs of the students. The teacher introduces and uses the mathematical language along with the story interpretation. Review of previous concepts, relationships, vocabulary and generalizations can be conducted at this time but it is essential to maintain a fast pace.

The inclusion of a mini-challenge is to clarify the major challenge, to exhibit the way in which the data is to be recorded or materials used, and to alert students to the possibilities that variations and relationships exist. Again this is done primarily by example.

The mini-challenge is conducted by first posing a question often within the story framework, but the measures to be used are substantially smaller numbers than those which will be given in the larger challenge. Feedback is in order and it should be given in much the same manner as mentioned above. As students find examples they should be displayed on the overhead or in whatever fashion they will be asked to maintain a record of their results during the major challenge. The conclusion of the launching phase is the launching of the major challenge, the task for the exploratory phase.

Discussion of Variations: Teachers were usually good about following the particular examples as outlined in the script. Some of them did not seem to feel comfortable in elaborating on the stories. At least they were not good, dramatic story tellers. They lacked sufficient "punch" so as to capitalize on the motivational and communicational benefits of the fantasy. One teacher suggested that his students prefered the mathematical language, but that was not the general reaction of teachers. Some teachers read the story comments from the script verbatum although it was not written with that expectation but only to suggest the basis of the story to the teacher.

Variations from these launching techniques sometimes had unfortunate effects. This was true for teacher 9 in activity 6. Students were to have been asked to build a solid block and measure it and that was to have been followed by several more examples.

The teacher simply held a small model in hand, a model too small to be seen by many students in the class, and pushed for an abstract rule for volume. The other examples were skipped entirely and the first example of the mini-challenge was given as the teacher continued to focus on the multiplication rule rather than the counting interpretation of the concept. The teacher never seemed to recognize the relationship between this problem and the major challenge, nor was this made clear in the script. In this case, there was no transition from the example to the challenge nor was the challenged offered in any story context. By the time the students were into the exploration phase they were very confused.

In activity 3, teacher 9 illustrated the consequences of failing to give the rapid-fire, yes/no feed back when introducing a concept. Instead, as this teacher walked around the room she paused to give personalized feedback to individuals. This left the rest of the class sitting and waiting for something to happen. It slowed the pace necessary to maintain student attention. This is not to say that such personalized feedback should never be given, but that it is inappropriate during this phase. This is the time that students should be depending on themselves, their peers and the correct examples to provide them with assistance. If assistance is needed, it can be given during the next phase.

Overall, teachers tended not to incorporate example problems that were not in the script. However, they also frequently omitted some of the items in the script. Whether this was due to the difficulty in following the script or to a deliberate choice on the part of the teachers was not known.

Inadequate launchings, whether due to deficiencies in the script or the execution of the script, reverberated through the subsequent phases. When the concepts or tasks were not made clear to the students prior to issuing the challenge, the teachers were deluged with requests for assistance or confronted with students not working on the challenge.

Exploration Phase

Model: The exploration phase was envisaged as a time when students worked on the major challenge of the activity. This was to be done in groups of 3 or 4. The teacher's role was to maintain student engagement on the challenge and to extend student involvement beyond the immediate task. To do this teachers were to circulate about the room noting student involvement and progress and to intervene when the situation warranted.

Maintaining moves were considered to be assisting, correcting or prodding. Offering assistance would entail helping the student to understand whatever had not been made clear during the launch. This is the appropriate time for giving individual attention.

Correcting moves are intended to be only brief comments directed to individuals whenever an error is detected on the student's recording sheet or from what they built or were saying. Students are expected to try and figure out for themselves or within their groups the cause of the errors. If they were unable to find their mistake, an assist would be timely.

Prodding is a technique for teachers to alert students to the fact that there may be more to the task than they realized. Students might prematurely believe they are finished with the challenge. This would put them back on the task.

Students who had completed the challenge were now in need of extending moves. The teacher was supplied with problems labeled as extra challenges to use with students evidencing a readiness for them.

All of the teaching moves described were intended to require minimal interchange so as to allow the teacher adequate freedom to continue to float about the class.

Discussion of Variations: A most obvious effect of an inadequate launch was the degree to which teachers became involved in assisting students to get started with the challenge. If there was a heavy press on the teacher's attention by students needing assistance, the teacher was not free to be sensitive to the need for prodding or extending moves.

At times teachers cut short the exploration before most of the students had gathered most of the data. This had serious effects on the summarization as they didn't have sufficient data to develop a pattern and they didn't want to discontinue their search for more examples. This can be seen in activity 2, where teacher 9 used only 11 minutes for the exploration while the other teachers used from 22 to 38 minutes for the same exploration. (see table 11).

As was modeled during the training sessions, teachers tended to make brief checking comments such as "I disagree with this response", or "Are you sure about this?". The teachers did not usually attempt to give explanations when these comments had been offered unless the need for further assistance became more obvious. Of course, to be able to make a correcting move required that the teacher be familiar enough with the content to recognize them, and teachers sometimes did not.

Prodding was not a commonly used technique by the teachers. Some teachers tried the techniques used in training such as to ask how many rectangles the students had found, or what the largest and smallest measures were. Observers frequently noted students in need of prodding moves which were never offered.

The students finished with the major challenge and ready for some extending moves were not always attended to by the teachers. Whether the teachers were able to reach them appeared to be dependent on the availability of the extra challenge. Thus, when the extra challenge was a package of blocks or a recording sheet and was readily available, they were more likely to be issued. But having to stop and give the extra challenge verbally to each student for whom it would be appropriate, was much harder for the teachers to do. Perhaps they could not remember the challenges.

Teachers use of extra challenges varied across activities and across teachers. For example, on activity 2, teachers 9 and 1 offered none while teacher 8 gave them to most all students.

Teachers seemed not to follow-up on student success with the extra challenges unless the results were discussed with the entire class.

Occasionally, teachers would pose extra challenges to the entire class during a summary. This was done comfortably by teacher 8 in activity 5 after the activity had gone quite smoothly and time was remaining before the bell. On the other hand, teacher 11 had no time during activity 1 to give extra challenges so he passed them out to all students at the beginning of the next day before beginning activity 2. As a result, many of the students focussed on the extra challenge rather than attend to the launch of the new activity.

Sometimes, when things were going smoothly, teachers were able to stand back and simply observe student behaviors. Even when they were busy, they were actually attending to many students. Teachers often commented on the ease with which they gathered more insights about individual performances in this fashion. Working with concrete materials made student progress or difficulties more evident than with just paper and pencil work and teachers recognized this.

Summarizing Phase

There are three main objectives to be accomplished during the summarizing phase. When most of the students have had an opportunity to gather most of the data, the teacher needs to obtain the attention of the whole class. Often this was best accomplished at the beginning of the next day.

The first task is to gather the data from the class. If the data is contributed by the students in a random fashion, the teacher can display it in an orderly format to allow patterns to emerge. Experimentation with different ordered displays might reveal different patterns.

As rules and equations are explicated by the class, the teacher can pose specific examples in order to verify the rules.

Discussion of timing: Timing was a most important variation in how teachers conducted the activities. The two aspects to timing are closely related. One is the pace at which things are progressing, which is reflected in the amount of time devoted to a phase, and the other is the choice of a particular time at which to initiate or terminate a phase.

To illustrate the way in which this variation is reflected in the instructional phases, the amount of time devoted to conducting each phase, as well as the total time, is listed in table   for each of the videotaped teachers in six activities. Some differences in times reflect the fact that teachers did not always carry out every step of the script while others were a consequence of how the presentation was made.

TABLE 11

Minutes Devoted to Activities By Instructional Phase and Teacher.

Activity		Teacher 9	1	8	11
2	Launch	25	12	42	14
	Explore	22	38	11	34
	Summary	13	11	10	8
	Total	60	61	63	56
3	Launch	18	7	14	6
	Explore	22	29	18	34
	Summary	16	11	10	5
	Total	56	47	42	51
5	Launch	26	18	36	} 48
	Explore	18	16	14	
	Summary	7	13	13	0
	Total	51	47	63	48
6	Launch	23	11	26	29
	Explore	29	} 32	18	26
	Summary	17	—	8	8
	Total	69	43	52	63
7	Launch	}16	}5	}25	}16
	Explore				
	Summary	27	20	25	17
	Total	43	25	50	33
8	Launch	}22	}33	}30	}6
	Explore				
	Summary	5	6	25	25
	Total	27	39	55	31

Although the total time devoted to each activity was not the same across teachers within an activity, the differences were slight for activity 2 (7 minute range), moderate for activities 3 and 5 (14 and 16 minute ranges) and a 26 minute range for activity 6.

The variations across phases within an activity were more extreme on activity 2 than any other. A 30 minute spread for launches and 26 minute spread for explorations are not insignificant as will be made clear in subsequent descriptions. On this activity the summaries were least variable. In one way the most significant variation in the summary phases was an absence of one such as found in activity 5 by teacher 11.

Closer examination of how teachers conducted each phase will help to explain some of the variations. For one thing, time available was a constraint. When time is running out the summary and exploration phases would be most affected. One reason that launches continued to vary was because the end of the class period was further removed and therefore, less constraining. Although teachers were told they could continue an activity into the next day, the more common practice was to finish an activity in one day.

Some of the time differences were not without consequence. For example, the small amount of time alotted for the exploration is activity 2 by teacher 9, was insufficient for students to complete the challenge which in turn meant that the summary was less effective. Why this teacher chose to begin the summary under these conditions can only be surmised. It was, after all, early in the unit which contained new material, required a new way of teaching and was being both observed and video taped. But this same teacher cut short the exploration phase on other occasions as well.

B. TEACHER PERFORMANCE IN THE ACTIVITIES

In Appendix C, specific and detailed responses of the teachers to the script are presented. These responses were gathered from observations by the co-investigators, by reviewing video tapes of the instruction, and by self-reports by teachers in their daily logs.

In this section we wish to describe in a more general tone, the more typical reactions of the 13 teachers to the individual activities.

Activity 1. The beginning of the unit provided anxious moments for many of the teachers. Many were not used to teaching their math classes as a total group, or having the students work together in small groups, or using concrete manipulatives such as the ceramic tiles and marked string. Many also found it cumbersome to follow the script and remain in contact with the class.

Some deficiencies in the script became obvious at once. There was an attempt to build in a way for the class to "discover" logically that a square was also a rectangle. Much detrimental time was spent in this task where all that was needed was for the teacher to tell them it was.

Also, the use of the strings turned out to be more bother and lead to more misconceptions than they were worth.

Teachers frequently displayed inconsistant use of the vocabulary particularly with regard to the edges. Several times length and width creeped in and tended to cloud the discussion.

Activity 2. Teachers tended to provide excessive review of previous work even when it was not called for in the script. They were usually impressed and surprised by the amount of variation in student performance and understanding.

Provision for the summary was left out of the script by mistake but the teachers usually provided it anyway. There were several examples of teachers wanting to focus on rules and generalizations to a greater degree than the students were interested.

Activity 3. The reactions were much the same as to activity 2. The spread of student responses was even more impressive.

Activity 4. This was the most effective activity. The teachers enjoyed it and felt confident in conducting it.

Activity 5. This activity required the introduction of new language to describe 3 dimensional objects and also to change the unit of measure of surface area from 1 square cm to 4 square cm. These technicalities helped to develop confusion in some classes where the teacher wasn't completely confident with the ideas.

Activity 6. The greatest challenge and need for teacher understanding of the nature of the mathematical task occurred in this activity when surface area is constrained and the task is to maximize the volume. In several cases that lack of understanding by the teacher resulted in greater confusion by students. While the script didn't call for analysis of the cost/pellet ratio, a few teachers provided some discussion of the idea.

Activity 7. As in activity 4, this one seemed easy to present and conduct. Several teachers found this to be a good time to refer back to the 40-mouse tape on the wall and discuss the mouse coat question.

Activity 8. As the culminating activity, several ideas converge in the growing cube problem. The script was very inadequate in preparing the teachers for the classroom dynamics which are present during the building of the 4, 5, and 6-cube. With advanced warning, several teachers were able to control and exploit the exercise. Usually the teachers were pressed for time and had to rush the finish. As a consequence, a rich discussion of the mouse and elephant questions didn't occur. There may have been follow-up discussions in some of the classes but we have no evidence.

> In the introduction to the script and in the training session the suggestion was made that the teachers might encourage the students to develop a written story about the mouse and elephant much like the Nuffield publication "How to Build a Pond". There was little response to this suggestion probably because of the pressure of time.

C. TEACHER JUDGMENT OF STUDENT PERFORMANCE

Teacher judgments of student performance were collected on a daily basis during the teaching of the unit by pre and post pupil sorts according to student performance in class and predictions of student performance on the final evaluation.

Comments about individual students whose performance in some way surprised the teachers were also gathered during the debriefing sessions held at the conclusion of instruction on the unit.

Teacher Agreement on Student Performance Before and After the Mouse and Elephant Unit: Teachers' sortings of students on the basis of their general performance in mathematics, taken before teaching the unit, were compared with their judgments of student performance on the unit. This was in no way a predictive situation and so the Gamma measure of association was computed. This statistic is a nonparametric test of association for categorical data which is ordered.

As can be seen from table 12 which lists the teachers in order of their measure of association, there was a wide range. The larger the γ the more consistently teachers perceived student performance on the Mouse and Elephant unit and their previous work in mathematics.

What this measure of association suggests is dubious, however. Teachers may have a high agreement because of having formed realistic judgments in the first place which were still valid or because they were more resistent to changing the original opinion.

What is clear is that some teachers viewed student performance differently on the two occasions. Although nothing conclusive can be said, some conjectures will be made on the basis of observation and teacher comments.

TABLE 12

Measure of Association (Gamma) between Teacher Judgment of Student Mathematics Performance Before and After Teaching Unit

Teacher	Gamma measure of association
11	.96
4, 6	.84
7	.75
10	.73
13	.70
1	.65
8	.59
5	.50
9, 12	.49
2	.41
3	-.07

Teacher 3, who showed almost no agreement (-.07), was teaching a "borrowed" class and had only known his students through reading classes or from the basketball program. Evidently, these experiences were not good predictors of student performance in mathematics.

Teachers 2, 8, 9 and 12 were definitely teaching in a manner counter to their usualy approach. Consequently, the contrasting style may have enabled them to perceive the students in different ways and on different dimensions. This might account for their somewhat lower measures of agreement.

At the other end of the spectrum were teachers who had tried to implement concrete experiences on previous occasions though not necessarily with the classes they were teaching (1, 3, 4, 7 and 11). While this would not explain the variations as they exist, it might contribute to them.

Teacher-Observer Agreement on Daily Student Progress: In the four classes being more closely observed and videotaped, both teachers and investigators were to have completed daily reports on students accomplishments. Due to unavoidable circumstances, teacher 8's class was never rated by the investigators and teacher 11's class had very few days for which both teacher and observer judgments were made. Also, the number of students in which the measures of association could be computed were usually less than the number of students in the class because of absences and the option in the rating scheme to say "no idea".

In any event, a conservative measure of association was figured for each day of available data. The Lambda (symmetric) test was a conservative measure because it failed to take into account the ordering of the categories: 'has it', 'sort of has it' and 'does not have it'. The measures do not reflect a high agreement between raters with statistics varying between 0 and .61 but with all but one being less than .5.

TABLE 13

Measures of Association (Lambda) between Teacher and Investigator. Daily Judgment of Student Progress

			number of ratings investigator total		
	Teacher 1	Lambda	higher	lower	total ratings
	Day 1	.33	2	2	13
	3	.14	5	4	23
	4	.20	5	2	18
	5	.43	3	1	20
	6	.06	8	1	16
	7	.43	0	3	13
	8	.31	5	1	22
Totals			28	14	125
	Teacher 9				
	Day 1	.61	13	1	25
	2	.47	3	2	20
	3	.29	4	3	18
	4	.25	0	9	24
	5	.06	3	7	23
	6	.42	4	3	23
	7	0	3	7	19
	8	.27	1	8	22
Totals			31	40	174

Table continued on next page.

		number of ratings investigator total		
Teacher 11	Lambda	higher	lower	total ratings
Day 1	.40	8	0	19
8	.31	2	4	20
9	.37	6	2	18
Totals		16	6	57

Of the 125 individual ratings available for teacher 1, 22% were instances in which the observer surpassed the rating of the teacher. But for teacher 9 this was reversed with 23% of the ratings of the investigator being below the teacher's. On three days of data on teacher 11, the investigator rated the students a bit higher (28% of 57).

All of this suggests some variation in how teachers and the investigators might perceive the progress of the students in learning the content as well as some difference across teachers. But, it should be remembered that the test was a conservative one, based on relatively few cases and no conclusions can be drawn.

Teacher Judgment of Student Performance of the Final Evaluation by Test Item: The effectiveness with which teachers were able to predict student performance on an item by item basis is expressed in table 14 . Since teachers were not required to make predictions in any specified manner so as not to force an unnatural categorization, their statements could not be coded in an identical manner. Consequently, both brief teacher comments and actual percents of students responding to the primary tasks of each problem are listed together.

A simple comparison of the teachers expectations for student performance on an item and the percent of students answering correctly suggest several tentative conclusions.

The over estimates of student performance by teachers were higher in the non-concrete items (3-6). The greater the abstraction of the problem, the less successful were the teachers in predicting student performance. In general, the tendency was for optomistic prediction by the teachers.

TABLE 14

Teacher Prediction and Student Performance* on the Final Evaluation

Teacher							
1	Teacher Prediction	no trouble	some errors	better than 2	work A > P	harder	struggle hard
	Student Performance	94	77	77	71	63	38
2		really well		40	75	75	70
		87	62	43	54	36	8
3		some mix up	ok tougher	ok tougher	not too good	ok	less than majority
		85	55	64	47	31	23
4		can do it	50	ok sharper kids	most ok	pretty well	won't do well
		67	36	72	30	26	6
5		really well	no trouble	not much trouble	not much trouble	lot of difficulty	trouble
		92	73	71	54	46	19
6		no problem	good	most all	50	80	few
		84	59	70	50	48	22
7		not much trouble	most do well	most	most	should	53
		92	66	86	45	61	4
8		very well	good	good	very well	good	good
		93	86	94	80	80	58
9		Ok	alright	few	25	46	20
		81	55	65	30	30	14
10		88	65	50	34	30	15
		90	78	78	52	44	34
11		58	55	54	45	50	42
		73	71	71	42	48	12
12		should get it	expect to	hope so	62	should	75
		84	71	71	54	52	25
13		no judgments available					

*Key on next page

*(percent of student success)
Item 1 - average of measuring and labeling area and perimeter
Item 2 - building to a fixed measure
Item 3 - finding variations with a fixed measure
Item 4 - finding area and perimeter
Item 5 - volume and surface area
Item 6 - volume and surface area

Teacher Judgment of Individual Student Performance of Final Evaluation: Teachers were asked to sort their students into four piles that would predict how well the students would perform on the final evaluation. Student test scores were then ordered and placed into groups of comparable size. Somers' d test, a non-parametric test used with categorical data, was used to provide a measure of the predictability of the teacher's ratings on the actual success of the students.

The results for most of the teachers were quite similar and ranging from .32 to .52. Teacher 10 is an older, experienced math teacher in a middle school and teacher 3 was not teaching his regular class and didn't know the students very well.

TABLE 15

Measures of Predictability (Somers' d) of students Final Evaluation Score from Teachers' Pupil Sorts

Teacher	Somers' d	Teacher	Somers' d
10	.62	1	.41
5	.52	8	.35
2	.50	4	.34
6	.47	9	.32
13	.46	7, 12	.27
11	.45	3	.06

"Surprises": What could never be adequately expressed in the statistical measures of associations were the comments teachers made about their "surprise" students. During the debriefing session following the last activity the recollections teachers recounted were most often expressed in a positive or negative vain. These were in no small way related to the opinions teachers held of the students' performance before the unit about the mouse and elephant was taught.

When comments were organized according to the positive aspects two distinctively different categories emerged.

A. First, were the students teachers had initially rated low on mathematics performance but whom teachers perceived as having been more involved and contributing to the lessons. In their remarks teachers described them as really getting turned on, even when they were normally not confident or was difficult to motivate. Or when students started out with some difficulty and eventually improved, teachers noticed. Enjoyment and success working with manipulatives was mentioned even when students had very poor reading and writing skills.

B. The second type of positive remarks were made about students with better initial ratings as to their mathematics performance. These comments seemed to be about something significant they contributed or were capable of doing such as having a rule or exceptional ideas, needing concrete experiences or simply doing better than expected.

Though affective comments were not solicited one was offered that could only be interpreted as positive, "It was the first time he smiled!"

D. The other category of negative remarks was about all types of students. The essence of these comments was that the students had simply done less than the teachers had expected; there was little to suggest exactly why the teachers had been disappointed.

The frequencies of teacher comments according to these different categories are listed in table 16. The bulk of all the "surprises" which included about 70% were positive. Roughly 50% were recounting teachers' pleasure with performance of their less able students. Being able to observe and interact with these students on tasks for which they could complete and compete was definitely significant to the participating teachers.

TABLE 16

Frequencies of Teacher Reported.
"Surprises" by Teacher* and
Nature of the Surprise

Nature of Surprise		Teacher												
		1	2	3	4	5	6	7	8	9	11	12	13	Total
Positive	A	2	3	3	2	5	3	5		2	1	2	3	31
	B	1		2	2				2		1	1	2	11
Negative	C	1			2			1				1	1	6
	D	2		3					3		2		1	11

*No data available on Teacher 10

Only teacher 8 had no remarks from this first category. This may have been due to the fact that this class consisted of a fairly competent group of seventh grade students. There may have been no students to report that fit this description.

Several teachers registered no negative surprises (2, 5, 6 and 9). Overally, these teachers also talked about fewer students from their classes. The teacher who recounted the most surprised was teacher 3, the same person who has been mentioned before as having been teaching a group of students unfamiliar at least with regard to their mathematics abilities.

Although on 6 students rated as very good were also described as encountering difficulties. Although few in number, there were also fewer top students overall. But, the remarks teachers made suggest that the criteria on which the initial judgments were made might be worth investigating.

D. PROJECT TEACHERS REACTIONS TO THE UNIT AFTER TEN MONTHS.

Question: What do you think of the mouse and elephant unit now?

1. Great, in the sense of going concrete to abstract. High student interest and neat ultimate challenge. Some problem in 1st and one other activity.

2. It gave me an opportunity to use concrete learning methods because of the resources brought in by M.S.U. Usually I teach with abstract learning methods because of time and material. I learned a great deal about students being able to solve equations and paper problems, but the students who did this well found difficulty with this work when used in real life!

3. Fun to teach. I agree with that approach to learning. The script is important for teachers who have little background in math (like myself) and it reduces preparation time. I enjoyed seeing how interested the students were in the program.

4. Great! Made the children think and discover. Important math concepts taught/presented in a novel way. Discovery approach. Children had a better understanding at the concepts through this unit. It was challenging but not frustrating to the children.

5. It was an excellent experience for myself and my class. It is a highly creative way to teach skills with manipulation and problem solving. It involved the students totally and was motivating for them. They loved it!

6. Very good, made the kids think. Very glad to see such great organization give so much freedom for the kids to think.

7. Excellent hands-on method for teaching mathematical concepts which are difficult for children to grasp. Especially surface area and volume.

8. I liked it, although I felt pushed to complete it on schedule. I could have used more time some days and would like to have had about 2 or 3 more days altogether.

9. I enjoyed it, so did the class. I'd like to do it again if the materials were available (tiles, blocks, etc.)

10. It was a good unit, better than most of mine and one that taught the students some valuable concepts. I hope to be using a modified version of it later this year. It was very worthwhile to me professionally.

11. Great, I would like to do it again this year. The second time around should be even better.

Question: Did the mouse and elephant unit require you to teach differently?

Teacher

1. I normally did not teach total group. But, rather instruct smaller groups. Also, the "cutting and pasting" is less likely to be used in my class in other phases of math.

2. Very much - see number 4.

3. Yes, in most instances I follow the math book.

4. Somewhat. I do try to use a discovery approach to most math concepts, but not to the extent that this unit required. The unit required "hands-on" materials which I rarely have used. The script made me stay right on the topic - at times I am apt to stray from the topic I am working on.

5. The unit required unusual teaching procedures in that I rarely teach math to the class as a total group. It also required me to adjust to observing the thinking of students as they manipulated the objects and evaluate their methods rather than evaluating through answers on a paper.

6. Yes, certainly. Made the students think and use their own discoveries. I was much more of a guide than a leader.

7. Yes, namely grouping students and involving all students with hands on materials. Also allowing students to explore all possible situations and results.

8. Yes. Most of our program is individualized. I've not done large group instruction for several years in math. I also seldom use the overhead projector in math except for games.

9. Yes. I used one large group, which I rarely do and I did it for 1½ hours a day instead of 45 minutes.

10. Yes. There were some major changes in emphasis - especially in offering extra challenges to advanced students.

11. Yes, my program is objective based, a totally individualized program. Soon after the first time the unit was taught, I began using Cuisinaire rods which now have become an integral part of my program.

Question: Did your experience with the unit effect how you now teach mathematics?

Teacher

1. Yes, especially on the occasions that I teach total group. Also, it helped refresh my concern for moving from concrete to abstract.

2. Yes. I use more of this method to supplement the abstract method. I find students challenged much more - some students need more concrete methods to realize the importance of knowing why it is necessary to know about math concepts.

3. Yes. I have used the hands-on approach and self-discovery with particular lessons related to Geometry and the metric system. I continue to use the mouse and elephant unit.

4. Yes. 1. Have used more "hands-on" materials this year - example: geoboards, used some of the mouse and elephant concepts with the geoboards. 2. Trying more the discovery approach in math teaching - why a process works as it does.

5. I have sought to include extra challenges in the math work and encourage students to think beyond the required assignment or problem. It stimulated me to develop similar activities to include in "station-center" type areas. (Also, I use the overhead a lot more).

6. It made me aware of just how creative and good at figuring things out they can be!

7. Yes. I have tried to use the ideas with other concepts. As an example, paper folding for teaching the following vocabulary:

   1. point
   2. line
   3. plane
   4. vertical
   5. horizontal
   6. etc.

8. I don't really think so.

9. Yes. I am using more manipulative material.

10. It's main benefit was that I'm now more comfortable providing students with a challenge, problem or puzzle and providing them with primarily "yes" or "no" responses to their questions. Also, I'm more aware of the process of beginning with concrete and working towards symbolic.

11. This past fall I spent 3-4 weeks with the large group using the rods to do +, -, x, ÷ equations. We also did some preliminary work with Chisanbop. All kids use the rods when working with factors, primes, GCF, LCM, reducing to lowest terms and unlike denominators.

III. CLASS DIFFERENCES

This study was not designed to make comparisons across classes and it would be inappropriate to do so. The classes were not in any way equalized before the study began other than to limit the grades. There was no attempt to establish the prescore which might enable one to look at analysis of covariance or difference scores.

Despite the inappropriateness of comparing classes, the ordering of the class averages bears mentioning as it reflects age and possibly school differences. Out of the 13 classes participating in this study all were 6th grade classes except for two, one seventh and one fifth grade class. As it turned out, on the final evaluation and the extra evaluation these two non-sixth grade classes provided the upper and lower boundaries of scores for the remainder of the 6th graders. As can be seen in table 17, class 8 was the 7th grade class and class 13 was the fifth grade.

This display of the class means certainly highlights the fact that the 6th grade is a very dynamic year in the cognitive development of the students.

Again, while no conclusions are intended to be drawn, we observe that classes 1, 10, 5 and 11 are all classes in suburban middle schools and classes 7 and 12 were platooned and taught by special math teachers. Classes 4, 9, 2, 3 and 6 were all self-contained classes in elementary schools.

TABLE 17

Distribution of Scores on Final Evaluation Ordered by Class Means

Score/Teacher	8	1	7	10	5	12	11	6	3	2	9	4	13	Total
21	4	2		1	3			2						12
20	6	2		4	1		1			1				15
19	5		6	1	2	1		2	2		2			21
18	3	4	4	3		3	2	1	2	1	1	1		25
17	6	2	1	1		1	1	2		1	1	1	2	19
16	1	5	3		2	2		3	1	1			1	19
15	2	3	4	2	3		3	1	2	2	2	3		27
14	3	3	2	2	2	6	2	3	2	5	2	2	1	35
13	1		3	2	1	1	3	2	3			1	1	18
12		1	3	1	4	1	3	1	2	1	2	1	1	21
11	2	2	1	2	4	2		1	2	2	1	3	1	23
10		1	5		2	3	4	2	1	2	4	1	1	26
9			3		2	1	2		3	2	2		4	19
8				1	1		3	3	5	3	1	2		19
7			1	2					3	1	2	4	4	17
6					1			4	1	1		2	2	11
5						1		2		1	2	1	3	9
4				1						1	1	2	1	6
3										1	1	2		4
2		1									1		1	3
1				1					1					2
Mean	17.5	15.4	14.1	14.1	13.7	13.5	12.6	12.5	11.3	11.2	10.6	9.8	9.2	12.9
S.D.	2.8	4.0	3.6	5.3	4.2	3.5	3.4	5.0	4.3	4.4	4.9	4.5	4.2	4.7

IV. DISCUSSION AND CONCLUSIONS

The Mouse and Elephant Project emerged out of a deep concern on the part of the investigators that adolescent children in school deserve to be exposed to good mathematics taught well. The mathematical ideas associated with changes related to growth form a kernal of important content which can be regarded as good mathematics and, learned, will serve the knower throughout a lifetime.

Our basic interest and motivation at the beginning of the project was to determine how much of the content of growth relationships we could reasonably expect sixth grade children to understand, and also, what are the characteristics of the teaching of this mathematics that could be considered to be most effective for these students.

Our conclusions concern the students, the teachers, the unit with its' activities and our research methodology.

Conclusions Relating to Students: The first of the two major questions which was intended to be answered by this study is:

> "To what extent can 6th grade students understand the nature of growth relationships?"

On the basis of the data gathered from the Individual Assessments, the Final Evaluation, the Post Evaluation, from classroom observations, and from reviews of video tapes, it becomes clear that:

> Conclusion 1. Students in our 6th grade classrooms were generally able to understand the concepts of area, perimeter, surface area, volume, and the variability relationships which were in the unit when those concepts and relationships were presented in a concrete form. The students did not understand growth relationships at the abstract level.

Conclusion 2. There is a fairly consistant order of difficulty in learning the concepts across all modes of representation; that order being from easiest to most difficult: area, volume, perimeter and surface area.

Conclusion 3. There was a consistant order of difficulty in modes of representation across all concepts and relationships with the concrete being easiest and pictoral and abstract modes being more difficult in that order.

Conclusion 4. There was a great diversity in measuring skill levels displayed by the students. However, more than 50% of the students relied on counting in nearly all circumstances.

In teaching the unit the teachers were to introduce these measuring concepts to the students in a concrete, exploratory manner. Because of time constraints they weren't given the luxery of taking time for review and practice.

Conclusion 5. With very few exceptions, the students responded positively and enthusiastically to the materials, the tasks, and the challenges of the activities. They enjoyed approaching higher order relationships using concrete skills.

Conclusion 6. Frequently students who did not perform well in their standard mathematics class, emerged as more solid performers in this unit.

Conclusions Related to Teachers: The second major question which we wished to answer in the project concerns the teachers.

> "To what extent can experienced middle grade teachers be prepared to teach the unit successfully?"

Conclusion 7. Despite the substantial variations in background both in training and experience, all of our teachers exhibited a need for guidance from the script and more training when using concrete manipulatives in the classroom in conjunction with total class problem solving tasks.

> Conclusion 8. Experience with the unit increased the teachers' sensitivity to the need for concrete experiences for children and also the nature and extent of the individual differences in the classroom.

There is wide spread common agreement that much of the mathematics being taught today throughout the nation is still in an individualized, objective-based, paper and pencil format. Even teachers who engage in whole class instruction tend not to use manipulative materials. One major reason for this probably stems from the fact that, with all the demands and pressures on teachers, such programs are easier to conduct. Most of our teachers in the project did not use manipulative materials in their regular math classes.

Much more effort is required to lead class discussions, provide challenges, and provide experiences with manipulative materials for a whole class in a problem solving mode. If we are going to expect teachers to teach good mathematics, such as growth relationships, and teach it well, as described in this unit, we need to find more effective ways of helping them to do so.

> Conclusion 9. Given a carefully developed script with clearly described phases of instruction, experienced mathematics teachers need explicit instruction in the roles and techniques for successfully conducting the phases of instruction.

Conclusions Regarding the Unit: As we entered the project we had an idea of the possible phases of instruction which an activity should possess, but we didn't realize how important a clear specification of those phases and the means to execute them might be. The words launch, explore and summarize occurred in the introduction of the script with brief descriptions of their characteristics (see appendix A). During the training program, there was a short one-half hour discussion of the phases during the third session.

The phases were also modeled by the investigators when introducing the activities to the teachers. The script did not clearly delineate the phases. After witnessing the unit being taught in 22 classes, (we have taught additional pilot classes since completing the project), we are aware the script must be much more explicit.

When the present script was written, we were not cognizant of the important characteristics of an effective launch, of the teachers' propensity to review, of the best places for review, of the characteristics of a good summary, and many other details. These became more clear particularly as we reviewed the video tapes of the classes. The next generation of the script may be written so as to avoid many of the ineffectual practices the teachers exhibited.

> Conclusion 10. Each activity must be revised to conform to the three phases of instruction in a much more explicit and complete fashion. This would include the specific characterization of the roles and techniques which are appropriate for the successful conduct of each phase.

The unit has been taught in fifth grade and in a recent pilot class in eighth grade. The fifth grade very seldom moved to abstract levels in the tasks and, although they enjoyed the participation, their learning was the lowest of all classes. (see table 17). On the other hand, the only eighth grade class so far has tended to reject both the manipulatives and the fantasy of the unit. Maybe another class in another circumstance would be more receptive.

> Conclusion 11. Grades six and seven are optimal grades for the introduction of the Mouse and Elephant Unit in its present form.

Conclusions Regarding Research Methology: This small, exploratory project attempted to cope with questions associated with content, teaching, and learning. In the real world of a sixth grade mathematics classroom, teachers, students and observers must contend with physical limitations, social dynamics, cognitive concerns, personality interactions and administrative necessities. This study attempted to explore, or otherwise accommodate to all of these dimensions.

Our use of video tapes turned out to be quite different than we had planned. Due to complications in scheduling, we were not able to use stimulated recall with either teachers or students. However, the tapes were very valuable.

> Conclusion 12. Our best use of video tapes was as tools for later analysis of the teacher and student behavior and interaction during the teaching.

Another potentially valuable use of the video tapes might be to create protocal materials for inservice and preservice training of teachers. As the phases of instruction become more clearly specified, positive and negative examples of the techniques and strategies associated with those phases could be isolated. We have in our possession as a residue of our project, more than 90% of the tapes from our project. However, these plans were never part of our original agreement with our project teachers so they couldn't be used without further negotiation.

While our conclusions about learning are drawn from data from the entire group of students in 13 classes, or else from data on all students in the 4 videoed classes, we were able to witness widely varying teaching patterns and behaviors even with the script. We must assume that different teaching will result in different learning in some important ways. With the development of a more refined script and a careful preparation period, we think we can remove some of the questionable variability and unproductive behavior in the teaching.

> Conclusion 13. A more careful study of the effects of teaching good mathematics well on the learning outcomes by students in grades six and seven can only be accomplished when the teaching has become standardized in several important ways.

Those ways the teaching of this unit need to be standardized in order to learn more about learning outcomes are the following:

1) The teacher must have a thorough understanding of the content of the unit.
2) The teacher must have a rapport with the class so as to be in control without the need for a strict authoritarian atmosphere.
3) The teacher must be able to develop and embellish a story in which the concepts and relationships assume a concrete meaning in a problem solving atmosphere.
4) The teacher must understand the model of instruction, its phases, roles, techniques, their purposes and interrelationships.
5) The teacher must be able to execute those roles and techniques in ways which provide for the orderly conduct of the activity.

If we begin with the set of principles which are listed on page 3 , those principles suggest the development of a model of instruction such as the model emerging from this project. That model requires a script which will represent the third generation of a script for this unit with the elaboration and specification based on the experience gained in this project.

If a teacher possessed the five characteristics described above, and chose to teach the unit using the revised script, what would be the effects on the teaching of the unit? What would be the nature of the conduct of the activities? What would be the cognitive and affective learning outcomes from the students?

The pursuit of the answers to these questions seem to be the most reasonable next steps.

APPENDIX A "SCRIPT FOR THE MOUSE AND ELEPHANT UNIT"
MISSING FROM DOCUMENT PRIOR TO ITS BEING SHIPPED
TO EDRS FOR FILMING.

Appendix A - Script for the Mouse and Elephant Unit

Appendix B - A Brief Description of the Project Teachers and their Classrooms

Appendix C - Teachers' Responses to the Script

Appendix D - Evaluation Instruments

APPENDIX B

A Brief Description of the Project Teachers and their Classrooms

Teacher 1 is a young man with four years of teaching experience in an upper middle class school near a large university. He was half of a two teacher team with 50 sixth grade students. The team used two large adjacent rooms with a flexible wall between them. The atmosphere was casual and comfortable. The floor was carpeted. The students worked at tables in groups of four. The class was videotaped.

Teacher 2 is a middle aged man with four years of teaching experience coming into teaching later than most persons. His school is in a middle class neighborhood in a large city but has students in K, 5 and 6 only due to desegregation practices. The classes are self-contained. The classroom atmosphere, for mathematics, is rather formal with an intense focus on standard performance and a high dependence on abstractions and rules. For this unit the class was seated in semicircles oriented toward the center front.

Teacher 3 is a man with ten years of experience teaching in a K-6 self-contained classroom school in a middle class neighborhood in a large city. The class he taught for the project was not his regular class but rather a less capable class of the teacher in the next classroom. Mathematics class for this teacher is usually highly book-and-rule oriented. The students sat in straight rows facing the front. This teacher is a former USMES teacher and also taught the mouse and elephant unit in a previous study.

Teacher 4 is a women with 15 years experience teaching in a small mixed-neighborhood school on the edge of a large city. The classes are self-contained. This teacher's regular class is fifth grade but she taught a class of 6th grade students for the project. Most had been in her class the previous year. This teacher is very highly regarded in the district as a specialist mathematics teacher. Her class sat in rows facing the front reducing peer interaction. The teacher was very carefully prepared and in complete control.

Teacher 5 is a young man with 3 years experience teaching in a university community middle school. He is half of a bilingual (spanish) team which has several chicano students. The school has a relaxed atmosphere. The class was arranged in a large square around the room and had a lot of peer interaction. The class met during the last period of the day and tended to be boisterous. The teacher was confident with the content of the unit and was often looking for interesting extensions.

Teacher 6 is a young woman with four years of teaching experience in a K-6 self-contained classroom school in a small rural town. The students in the school are relatively unsophisticated and the school atmosphere is friendly, orderly and comfortable. The students worked eagerly in small groups around tables. The teacher was a little anxious but intent on doing a good job.

Teacher 7 is a man with nine years of experience in a large K-6 neighborhood school in a large city. The teacher was part of a 4-teacher team meeting in a large team room. The class sat in semicircles but with a lot of peer interaction. The teacher was formerly an USMES teacher.

Teacher 8 was a woman with 19 years of experience as a junior high mathematics teacher now teaching in a grade 6-7 middle school. The community is rural-suburban. The class is a selected group of seventh grade students who have previously been successful in mathematics. The class worked in groups of four in a format unfamiliar to the teacher. The class was video-taped.

Teacher 9 is a woman with 12 years experience teaching in a K, 5, 6 school in a middle class neighborhood in a large city. The classes are self-contained. There were a few advanced fifth grade students in the sixth grade class. The students worked in small groups. The teacher was unaccustomed to stand-up total class teaching in mathematics as her class usually was individualized. The class was video-taped.

Teacher 10 is a woman with 15 years experience as a junior high mathematics teacher. Her 6th grade class was in a grades 6-8 middle school in a suburban bedroom community. The class met in a large science lab with permanent lab tables where students worked in groups of four. There was a lot of interaction and high interest among the students.

Teacher 11 is a highly trained young man with 6 years of experience in a grades 6-8 junior high school in a rural-suburban community. The class met the last hour of the day and was fairly chaotic. The students worked around 6 tables in groups of 3-6 with a lot of interaction. The class was video-taped.

Teacher 12 is a woman with 13 years of experience now teaching in a K, 5, 6 school in which the 6th grade students are platooned. This teacher teaches 3 mathematics classes each morning. She is a former USMES teacher and somewhat specialized in mathematics. The students worked in clusters of 3 or 4 with healthy peer interaction. The school atmosphere was tense with a lot of teacher-administrator hostility.

Teacher 13 is a woman with 10 years experience and teaching half-days in a self-contained K-6 school in a lower class neighborhood in a large city. The other half of her day was spent at the university as a research intern. She is a former USMES teacher and taught the mouse and elephant unit during the previous year in another project. Her students were in the fifth grade.

APPENDIX C

Teachers' Responses to the Script

The comments which follow for teachers 1, 8, 9 and 11 represent a distillation of observer's notes and a careful review of the video tapes of the classrooms shortly after the teaching occurred.

The reactions of the other teachers were gathered from the self-reports in the teacher's logs.

When analyzing the responses, the reader should keep the following details in mind:

a) This was the first video experience for each of the teachers.

b) The first video class (teacher 9) was interrupted by seven consecutive snow days and was very rushed to complete the unit on schedule.

c) Teacher 8 was teaching a group of selected 7th grade students.

d) Teacher 11's class met the final hour of the school day. Ten minutes before the final bell, buses lined up immediately outside the classroom windows.

ACTIVITY 1

Teacher 9

Items 1-7 - Consumed 22 minutes. Much time spent trying to convince class that a square is a special rectangle. Much drag time and in attention.

Items 8-13 - The word "area" was extracted after 26 minutes. The word "perimeter" was written on chalkboard at 48 minutes. The students often counted the marks on the string rather than segments.

Items 14-16 - Skipped due to lack of time remaining.

Items 17-18 - During last 2 minutes the teacher distributed extra challenges to nearly every student but no recording sheets.

Teacher 1

Items 1-7 - Consumed 15 minutes. Many students knew word "area. Teacher suggested students consider covering floor of large room. Some students thought about strips.

Teacher 1 (cont.)

Items 8-13 - Teacher omitted items 11 and 12 and had to return to them after item 16. Many students knew "perimeter". Thirty minutes had been used so far.

Items 14-16 - Teacher omitted discussion of edges and had to return to it at this time. Students were eager to get the work sheet to provide them with something to do.

Item 17 - Many students wanted the extra challenges.

Item 18 - Teacher reviewed the difference between area and perimeter and pushed for generality. After the lesson he felt uncomfortable about it. He felt it was not smooth and the students were not challenged.

Teacher 8

Items 1-7 - Teacher proceeded smoothly through introduction in 17 minutes. The material was easy for the selected 7th grade students.

Items 8-13 - Item 10 was omitted as teacher felt it wasn't needed. Perimeter was defined after 23 minutes.

Items 14-16 - Taught smoothly and easy for students.

Item 17 - All students asked for extra challenges and worked very enthusiastically.

Item 18 - Summary not neccessary. Students were eager to know the correct answers to the extra challenges.

Teacher 11

Items 1-7 - Teacher followed script well but quickly and consumed 13 minutes.

Items 8-13 - Teacher focused on bottom edge and side edge carefully but didn't stress the word perimeter. Item 12b was skipped. At Item 13 the class attention began to dissapate. Teacher then skipped 13a, b, c, 14 and 15 to move to 16. Total time - 22 minutes.

Item 16 - Many students were asking what perimeter and area meant.

Item 17 - Teacher didn't distribute any extra challenges nor have any time for summary.

(It should be noted again that this class was at the end of the day and most of the students rode school buses.)

Other Teachers (teacher numbers are listed in parenthesis)

Item 2 - Students were not clear what was wanted. Some made rectangles with open centers. (6, 12)

Item 3 - There was a lot of confusion about the definition of a rectangle. Most students thought a square was not a rectangle. (5, 7, 10, 12)

Item 7b - Many students already knew the name for area. (4, 6, 7, 12)

Item 10 - Verbalization was difficult for many students. (3, 4)

Item 12a - Many students counted the knots in the string rather than the segments. (3, 4, 7)

Item 13 - Most of the students did not know perimeter. (7, 12)

Item 17 - Most students wanted extra challenges. (3, 4, 6, 7, 10, 12)

ACTIVITY 2

Teacher 9

Items 1-3 - Teacher used 42 minutes to review, issue mini challenge for 12 tiles and illustrate cut-out models. The result was a great amount of idle time and restlessness among the students.

Items 4-6 - The class was given 11 minutes to explore rectangles with area 24. Many were not able to finish in that time.

Item 7 - Summary was inadvertantly left out of the script. Teacher conducted summary anyway for 9½ minutes. Because the students hadn't finished their exploration, there was only mixed attention.

Teacher 1

Items 1-3 - Teacher used 23 minutes for launching phase. In item 2 he focused on edges but forgot perimeter.

Items 4-6 - Class worked well during exploration for 24 minutes. Teacher worked with individuals. He didn't provide any extra challenges.

Item 7 - Summary used 13 minutes. He was able to extract the complete pattern and drew out some rules for calculating perimeter.

Teacher 8

Items 1-2 - Quick review and launch required 7 minutes and went well.

Item 3 - Delayed until after item 5.

Teacher 8 (cont.)

Items 4-6 - Students participated well. Nearly all were given extra challenge.

Teacher 11

Items 1-3 - Teacher began by giving extra challenge from Activity 1 to those who didn't get one yesterday. Some of those students worked on the challenge rather than attend to the review. Later part of introduction worked well and was completed in 14 minutes.

Items 4-6 - Students cut out the models but teacher omitted items 5 and 6b. Thus, perimeter was ignored. Summary sheets were distributed just as class was leaving. Summary was conducted during the first 22 minutes of the next day.

Other Teachers

Items 1, 2a, b - Generally good reaction. (4, 5, 6, 12)

Items 4-5 - Great variation in students understanding became clear very quickly. (2, 4, 5, 6, 12)

Item 6c - Approximately one third of students were given the extra challenges. (2, 3, 5, 7, 10, 12)

Summary - (Item 7 which was omitted from script). Several teachers conducted discussions which elicited rules and relationships. (e.g. when it's stretched out you have a larger perimeter.) (2, 3, 6)

ACTIVITY 3

Teacher 9

Items 1-3 - Teacher interchanged 2 and 3. The results were not recorded nor were the rotations of rectangles shown. Launching consumed 14 minutes. Having students come to overhead to display was time consuming and students lost attention.

Items 4-5 - During 18 minutes of exploration the students would find an example, then wait with hands in air until teacher checked to see if it was correct.

Item 6 - Summary began before students had completed their exploration. As a consequence the students didn't pay attention but continued to work or play.

Teacher 9 (cont.)

Items 7-8 - The following day the teacher introduced tic-tac-toe but used a very large grid rather than a 5 X 5, so there was no contest involved. Then the parabola was plotted. The entire exercise consumed 17 minutes.

Teacher 1

Items 1-3 - Teacher used 10 minutes to launch activity and placed excessive emphasis on rules for perimeter and area. As a result, there was a lot of messing around.

Items 4-5 - Exploration used 22 minutes. Teacher moved about very effectively and distributed some extra challenges.

Item 6 - Teacher extracted unordered data, then ordered it and looked for patterns. All the rules for perimeter and area emerged.

Item 7 - Omitted because students knew how to graph points.

Item 8 - Well received.

Teacher 8

Items 1-3 - Teacher used 18 minutes to launch the activity. Results from Activity 2 were reviewed and extra challenge answers were discussed.

Items 4-5 - Exploration consumed 29 minutes.

Item 6 - Eleven minutes on the following day. Developed a good display of the cutouts in a pattern. There was some discussion of the extra challenge results.

Item 7 - Played on 10 X 10 grid first time, corrected second time.

Item 8 - Graph developed well as a parabola.

Teacher 11

Items 1-3 - Teacher omits item 2 and completes launch in 6 minutes. He emphasized the edges and perimeter by high lighting the segments on the overhead.

Items 4-5 - After 7 minutes of exploration, students put work away and were prepared to leave class 4 minutes early. The next day he re-issued the challenge after 6 minutes of review. Students worked well for 29 minutes.

Item 6 - Summary consumed 5 minutes until time ran out. It was very effective.

Teacher 11 (cont.)

Item 8 - Students knew how to graph so teacher began the next day graphing the parabola. Students were very interested. Consumed 7 minutes.

Other Teachers

Item 1 - Some of the teachers felt the students were confined by the banquet table metaphor and defined perimeter directly. (3)

Items 2, 3a, b - Generally good response. Provided good review of the definitions of area, perimeter, edge. (3, 6, 10)

Items 4, 5a, b - Wide variety of response. Some "zip through" while others are in tears. (4, 7, 10)

Items 6a, b, c - Patterns began to emerge in many of the classes. (2, 4, 5, 6, 7, 12)

Items 7, 8, 9 - Varied and generally good response to the graphing activities. (3, 4, 6, 7, 12)

ACTIVITY 4

Teacher 9

Items 1-3 - Introduced quickly and effectively. Students respond well. They work on block B for 13 minutes.

Items 4-6 - Students work steadily. Some extra challenges were dispensed. Effective 5 minute summary at end of class.

Teacher 1

Items 1-3 - Students respond well for 8 minutes.

Items 4-5 - Continued to work well on other blocks for 23 minutes. Teacher passed out some extra challenges as "exotic food".

Item 6 - Sixteen minutes of summary. Students couldn't provide much data without summary sheets. Teacher then began discussion of surface area and its measure.

Teacher 8

Items 1-3 - Teacher confused the story at first but then straightened it out and it went well. Students worked on block B for 20 minutes.

Teacher 8 (cont.)

Items 4-5 - Other blocks were used last few minutes and extended to next day for 40 minutes.

Item 6 - Eight minutes of summary went well. Students learning that different shaped jackets for the same block should cost the same.

Teacher 11

Teacher conducted a five minute discussion of the Mouse/Elephant question including displaying the mouse tape and obtaining guesses.

Items 1-3 - Teacher introduced an embellished story very clearly.

Items 4-5 - After ten minutes the other blocks were passed out. Students were more interested in going to other blocks than finding variations for same block.

Item 3-D - Next day teacher asked for varieties of jackets for block B. Many students interested for 12 minutes.

Teacher passed out extra challenge for activity 4, 5 and 6 which is a picture of 3-D blocks and was premature.

Other Teachers

Item 2 - Worked well. Some children try to wrap the block like a package with overlap. (4)

Item 4 - One student with eye problem had great problems. (7)

In General - The most successful activity. (2, 4, 7, 10)

ACTIVITY 5

Teacher 9

Items 1-7 - No challenge was issued to class. As teacher checked individuals she moved them on to 24 cubes but did not mention maximum or minimum. During the remaining 25 minutes students continued to work only after the teacher checked their work. Otherwise they just sat or played.

Item 8 - Reviews results with 12 blocks but related volume to formula and not to number of cubes. Language confused. Teacher ignores maximum and minimum. Asks for results for 24 but students have very few results. Next day teacher asks for summary of 12 blocks but ignored the maximum and minimum costs. Then asked for results for 24. Very few students had results. Time - 14 minutes.

Teacher 1

Items 1-4 - Students recognized need for third dimension very quickly. Teacher tended to talk through the challenge with the class rather than pushing them to explore and discover their own redundancies. Finally, the students focused on edges well. Time - 18 minutes.

Item 5 - Class discussion went well but the students weren't asked to build. Teacher asked how prime numbers related to the problem which seemed to serve as a distractor at this point. Time - 8 minutes.

Items 6-7 - Skipped.

Item 8 - Challenge was issued with excessive directions from teacher. After five minutes he had to explain surface area to class again. Many students were off tack. Some students could have used extra challenges but teacher was too busy giving individual help. Time - 18 minutes.

Item 9 - Teacher gathered data from students but didn't accept redundancies. Pattern emerged that the product of the edges was the volume. Cheapest was "squatty". Many students not attending. Time - 8 minutes.

Teacher 8

Items 1-7 - Went rather smoothly. Students caught on quickly to word "faces". Time - 18 minutes.

Item 8 - 24 block challenge. Whole class worked well and seemed to understand maximum and minimum costs. Time - 16 minutes.

Item 9 - Summary complete in 6 minutes. Teacher gave whole class challenge for 100 cubes. Maximum price came very quickly. Groups worked on minimum cost and got answer of 130 in 7 minutes before class ended.

Teacher conducted a 3 minute review the next day.

Teacher 11

Items 1-3 - Teacher began activity with only 14 minutes to go in the last period of the day. Built on A, B and C for 8 minutes.

Teacher then asked students to build on A, B and C with 24 cubes and record on green sheet. This was a total departure from the script and resulted in a chaotic situation for last 5 minutes.

Teacher 11 (cont.)

Item 5 - Next day. Costs for 1, 2, 3 and 4 cubes went very well. Students saw 2 different costs for 4 cubes.

Items 6-7 - Skipped.

Item 8 - Challenged with 24 cubes. Teacher forgot he had passed out the summary sheets the previous day so the remainder of the period remained in confusion with the observer becoming another teacher. There was no time for a summary.

Other Teachers

Items 2-4 - Students frequently needed playtime with the cubes. Keep colors uniform for each student to discourage trading. (4, 7, 10, 12)

Item 5 - Students frequently draw analog between surface area and perimeter and between area and volume. (3)

Item 8 - Several teachers skipped the problem with 12 cubes and went directly to 24. (4, 6, 7, 10)

Item 9 - Several teachers did not have opportunity to issue extra challenges. (2, 3, 10)

ACTIVITY 6

Teacher 9

Item 1 - Teacher pushed students to the abstract rule for volume without letting them build the blocks.

Item 2 - Skipped.

Item 5 - Introduced 1 X 14 X 1 for no particular purpose.

Item 3 - Teacher tried but continued to focus on multiplication rule. Time since beginning item 1 is 20 minutes.

Item 4 - Teacher summarized and gave answers to students. 6 minutes.

Item 6 - Without relating to item 5, teacher posed challenge for $58. There was no story embellishment. Students very confused. No one obtained best answer. Time - 18 minutes.

Item 7 - A few of the students had some data but no solution was reached. Time - 8 minutes. Teacher summarized for 5 minutes the next day and gave the answers to the students.

Teacher 1

Items 1-4 - Introduction proceeded rather well with mixed attention. Multiplication rule emerged in 15 minutes.

Items 5-6 - Introduced challenge. Students were confused. The story was not clear enough to make the challenge clear. Time - 32 minutes.

Item 7 - Asked for rules for surface area. Some students gave formula for volume. Others say "just count it". Teacher then received formula for volume. Time - 7 minutes.

Teacher 8

Items 1-4 - Proceeded smoothly. Time - 14 minutes.

Items 5-6 - Introduced challenge of $58 effectively. Asked for cost/pellet. Students worked 32 minutes.

Item 7 - Review and summarize next day for 3 minutes. Students solved the problem.

Teacher 11

Item 2 - Teacher began by asking for class to build specific blocks and asks for cost. Went very well until teacher gave attention to one table and rest of class lagged. After 7 minutes he passed out the green sheet.

Item 1 - Asked cost of first one. Time - 5 minutes.

Items 3-4 - Skipped.

Items 5-6 - Students did well with 1 X 14 X 1. Teacher asks to send 14 for less money. After 5 minutes, solution is found to the $46. Teacher then realized his error and issued correct challenge. Students work remaining 20 minutes with no summary.

Next day, teacher reissued challenge and students worked for 19 minutes.

Item 7 - Made complete listing on overhead and found best solution. Time - 7 minutes.

Other Teachers

Items 1-3 - Most students recognized rule for volume quickly. (all)

Other Teachers (cont.)

Items 5-6 - Many students had difficulty. There is a lot of room for exploration in this activity. (3, 5, 7, 10)

Item 7 - A few teachers attempted to introduce the idea of cost/pellet but there was little success. (12)

ACTIVITY 7

Teacher 9

Items 1-5 - Went smooth and according to script. Time - 25 minutes.

Item 7 - Summarized answers from students. Extensive use of rules by teacher. Time - 25 minutes.

Item 9, 11 - Lot of involvement by students. Teacher had difficulty displaying the squares effectively on the overhead. Time - 10 minutes. This was followed by a very brief discussion of the mouse coat problem.

Teacher 1

Items 1-5 - Went smoothly. Teacher used extra challenges very effectively. Time - 16 minutes.

Item 7 - Gathered data well. Teacher tended to focus on rules which many were ready. Introduced exponents. Time - 6 minutes.

Items 8-11 - All from overhead projector. Again, the focus on rules. Only a few really participating. Time - 10 minutes.

Item 12 - Last 6 minutes. "How many 1-squares in a 240-square?" Good move by teacher. No students correct.

Teacher 8

Items 1-5 - Students speed through in five minutes.

Items 7-8 - Good review and summary in 8 minutes.

Items 9-12 - Much good response. Time - 12 minutes.

Teacher 11

Items 1-5 - During last 10 minutes of period students were introduced to idea and told to cut models. Next day is an a.m. class again. Students work well for 14 minutes.

Item 7 - Teacher summarizes results in 2 minutes.

Teacher 11 (cont.)

Items 8-11 - Teacher moves around room posing questions verbally for 10 minutes. Many students are involved.

Item 12 - Poses M/E question and explores for 7 minutes.

Other Teachers

Item 5 - Most students wanted to cut out squares of many different sizes. (5, 6, 10, 12)

Items 9-11 - It is desirable to have many examples of larger squares available to illustrate these relationships. (4)

Item 12 - Several teachers referred back to the 40-mouse tape at this time. (4, 12)

ACTIVITY 8

Teacher 9

Items 1-5 - Some students need a lot of individual help. Time - 30 minutes.

Item 6 - Focus is on rules. Those students who hadn't gathered their data were not involved. Time - 12 minutes.

Item 7 - Teacher set up the problem and had each student find the answer. Time - 13 minutes.

Teacher 1

Items 1-5 - Teacher attempted a 5 minute review of M/E question, guess my rule for linear equations. It was attended poorly.

Then assigned class to build 1, 2, 3 cubes. Summarized the 3-cube. Time - 14 minutes.

Item 5c - Encouraged building of 4, 5, 6 cubes for 8 minutes. Then he had them put cubes away for summary.

Items 6-7 - Summarized findings for 1-6 cubes. Pulled rules for surface area, volume and number of mouse coats. Time - 5 minutes.

Teacher 8

Items 1-5 - Time - 33 minutes.

Item - Good summary. Time - 6 minutes.

Item 7 - Some students solve M/E completely.

Teacher 11

Items 1,2,5 - During last 6 minutes of class he asks for surface area of each.

Next day he checks on 1 cube and 2 cubes then goes to recording sheet to solve the M/E questions.

Item 6 - Students working at desks with cubes and recording sheets. Time - 25 minutes.

Item 7 - Teacher tried to summarize M/E questions. Students were still working or playing with cubes so there was little attention.

Other Teachers

Items 2-3 - This activity caught many students by surprise with a lot of initial misunderstanding. (2, 7, 10, 12)

Item 5 - By this time the students usually saw the pattern quickly. Careful supervision was necessary to get the 4, 5, and 6 cubes built in a cooperative manner. (3, 6)

Item 7 - All classes solved the problem together by generalizing from the pattern. Teachers realized that understanding was very spotty and many students didn't understand the answer.

APPENDIX D

Evaluation Instruments

Direction for Pupil Sorts -- You may make as many piles on a given as you desire.

Initial Sort to be done January 18, 1978

A. Sort your students into piles on the basis of their mathematics performance.

Describe your piles.

B. What changes would you make, if any, if you had been asked to sort on the basis of their mathematics potential?

Describe your piles.

C. Sort your students into piles on the basis of their involvement (ccoperation)(behavior) in mathematics class.

Describe your piles.

Final Sort to be done at the completion of the Mouse and Elephant Unit.

Repeat A,B and C above only restrict the sort to the unit.

On the day of the Final Evaluation for the students, the teachers will be asked how they expect the students to do on the various problems of the test.

Card Sort Descriptions teacher ________

Sort # ________ # of piles ________ date ________

pile # description

Sort # ________ # of piles ________ date ________

pile # description

Sort # ________ # of piles ________ date ________

pile # description

D-3

MOUSE & ELEPHANT: Student Roster Card Sort Data

Teacher ______________ # ______

Student Code	Roster	1	2	3	4	5	6	7	8	9	10
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											
12											
13											
14											
15											
16											
17											
18											
19											
20											
21											
22											
23											
24											
25											
26											

Directions for the Daily Report on Individual Students

A. Each student will be listed (first name, last initial) by the appropriate code - roster.

B. . Each daily lesson for each activity worked on during the lesson will require that this sheet be filled out. Usually, you will have only one activity on a day and at most two activities.

C. After each student check the appropriate column of H, S, D, or N according to the following:

H = Student has it -- has acquired knowledge and skills of the activity.

S = Student sort of has it -- not sufficiently, but has learned something.

D = Student does not have it -- is essentially out of it.

N = No idea -- you have not noticed anything about the individual's performance.

This is all your best judgment according to what you can recall about the lesson. You will be able to do a more accurate job if you responded to this as quickly as possible after teaching the lesson.

D. Write in more detailed descriptions about the individuals according to the following:

i) Write C for counting / O for organized counting / R for rule application to indicate the measuring skill level you were aware students were using.

ii) Specify misconceptions or difficulties you saw evidenced.

iii) Specify exceptional or interesting performances or insights you were aware students made.

E. Indicate if and what extra challenges were offered to which students.

MOUSE & ELEPHANT; Daily Report on Individual Students

TCHR CODE______

Activity ______ Day ______

Student Code	Roster	Check Tchr Judgment				Desciptions: skill level (C, O, R), difficulties, interesting bits.	Extra Challenge
		H	S	D	N		
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
1							
12							
13							
14							
15							
16							
17							
18							
19							
20							
21							
22							
23							
24							
25							
26							

D-6

MOUSE & ELEPHANT

Classroom Observation Form
Class Interaction

tchr ______

activity ______

observer ______ time began ______; ended ______ day ______

page ______ video tape ______

Questions	Responses	Probe Check

D-7

MOUSE & ELEPHANT

Classroom Observation Form
Work Time

tchr_____

activity _____

day_____

Observer________

time __________

Groups Individuals	Mid Rng	Skill Level	Difficulties	Insights	Participation
1					
2					
3					
4					
5					
6					
7					

D-8

Mouse & Elephant

Daily Record - Video

Observer ___________

tape #___________

date__________ pg _____

tchr __________

activity________;day _____

TIME	TAPE #	QUART	COMMENTS:

MOUSE & ELEPHANT

Daily Log of Recording Information

teacher ______

activity ______

day ______

date ________ time of lesson: began ______

ended ______

video tapes used:

number	tape numbers/ quarters
----	------------
-----	------------
----	------------

students absent:

any unusual circumstances/happenings:

camera observer ______________ classroom observer ______________

students receiving individual evaluations:

evaluator ______	evaluator ________
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.

Directions for Daily Teacher Log Reports

A. This is a record of what happens as you teach the unit. Each day for each activity covered you need to complete the teacher log.

B. In the left column write the number or numbers of the script which you followed and in the right column tell something about how the students responded. Split the script into "chunks" of script numbers (i.e., 3-7 versus 9) to accomodate shifts in the student responses or participation worth mentioning.

If you can elaborate on how you handled a particular portion of the script write it in the middle column after the script numbers.

C. Since you will probably have deviations from the script we would like a record of those deviations. Place a check (√) in the script # column and then describe the modification and why you deviated.

Also give a description of student response and participation.

Omissions will be obvious (number not included) and the location of the deviations should be apparent from the script numbers before and after.

MOUSE & ELEPHANT

TEACHER LOG

Teacher # ________ Activity _______ Day _____ page ____

Script #	Description of Modifications of Script	Description of Student Response & Participation

Individual Evaluation

RECTANGLES
Activities 1,2&3

1. (Show rectangle on pink paper.) What is this shape?

Please cover it with tiles. What measures can you give?

(If need be ask for perimeter and area and edges.)

(Check what interpretation children have for the measures they do give. Get the name and the interpretation.)

2. (Show rectangle cut out of grid paper.) Can you find the area and perimeter of this rectangle?

(If they count ask: Can you do it a faster way?)

3. Please build me a rectangle with an area of 16. What is its perimeter?

Can you make me another rectangle with area of 16 which has a longer perimeter than yours? (If they say no, stop and go to # 4.)
(If yes, ask them to show you.)
Can you make me another rectangle with area of 16 which has a shorter perimeter than your first one? (might need to give their number)

(If yes, ask them to show you.)

4. Please build me a rectangle with a perimeter of 14. What is its area?

Can you make me another rectangle with perimeter of 14 which has a larger area than yours? If they say no, stop. If they say yes, ask them to show you.)
Can you make me another rectangle with perimeter of 14 which has a smaller area than your first one? (might need to give their number)

(If yes, ask to show.)

Individual______________ #___ (Table ___) Teacher ______

day_____ activity _______ interviewer_____

RECTANGLES: Activities 1,2,3

Asked:

1. Shape______________

	measures:	method:	prompts:	interp:
edges: bottom side				
area				
perimeter				

Special Comments:

2.

	measures:	1st method	2nd method (if any)
area			
perimeter			

Special Comments:

3. Area 16; perimeter: measure_________ method______________

	initial response	other tries	concluded
built			
longer?			
shorter?			

Special Comments:

4. Perimeter 14.

	initial response	other tries	concluded
built			
larger?			
smaller?			

Special Comments:

Individual Evaluation

SOLID BLOCKS
Activities 5&6

1. Please build me a solid block with the following dimensions:

   3 on the bottom front
   2 on the bottom side
   3 high

2. What measures can you give me? (Use solid block built in # 1)

   (If need be ask for the surface area. If that does not make sense then ask for the cost of the space armor at $1 per square.)

   (If need be ask for the volume. If that does not make sense then ask how many food pellets were in the package.)

3. (Show picture A) Can you find the surface area? the volume?

   (No hints.) (Have them find the measures.)

4. (Show picture B) All the solid blocks in this picture have the same volume, but as you can see they are stacked differently.

   Will they have the same or different surface areas (space armor) ?
   (If say SAME, stop. If say DIFFERENT, ask:)
   Which shape will have the largest surface area? (cost the most to wrap)

   Which shape will have the smallest surface area? (cost the least to wrap)

5. (Show a 1 by 1 by 6) This cost $26 to package it for its flight to mars. It costs $1 per square.

   How many food pellets (cubes) are in this package?

   Could you package more than 6 food pellets for $26 or less?

   (If they say no, stop.)

   (If they say yes, ask: Show me how.)

Individual____________________#___ (Table ___) Teacher _______

day______ activity ________ interviewer_____

RECTANGLES: Activities 1,2,3

Asked:

1. Shape____________________

	measures:	method:	prompts:	interp:
edges: bottom side				
area				
perimeter				

Special Comments:

2.

	measures:	1st method	2nd method (if any)
area			
perimeter			

Special Comments:

3. Area 16; perimeter: measure___________ method___________________

	initial response	other tries	concluded
built			
longer?			
shorter?			

Special Comments:

4. Perimeter 14.

	initial response	other tries	concluded
built			
larger?			
smaller?			

Special Comments:

MOUSE & ELEPHANT
EVALUATION

____
name

teacher # _____

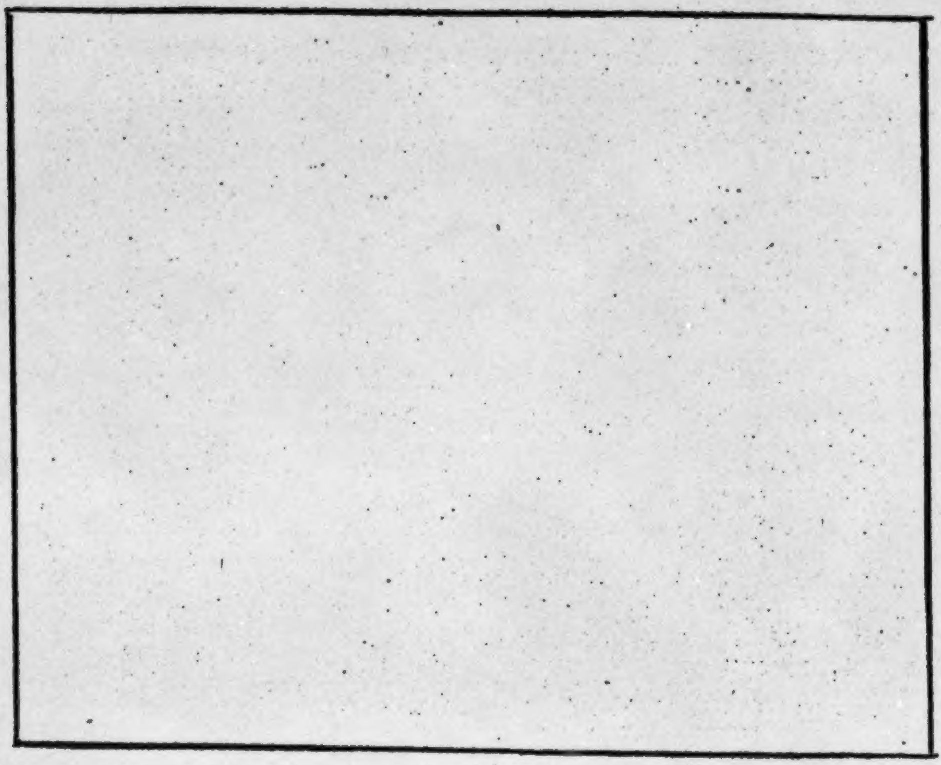
1. What do we call the figure drawn below? ____________________

Use your tiles to help find:

edges: front__________ side ________

area __________

perimeter _____________



If you need to ask what area or perimeter mean -- raise your hand.

mouse & elephant
evaluation

2. A.Use your tiles to make a rectangle with a perimeter of 20.

What are its edges? side ______ front ______

What is its area?_________

B. Can you make a rectangle with a perimeter of 20 that has a larger area than the one you have just built? ________

If you can, what are the edges? side______ front _______

What is the area? __________

C. Can you make a rectangle with a perimeter of 20 that has a smaller area than the one you first built (part A)? ________

If you can, what are the edges? side ______ front ______

What is the area? _______

mouse & elephant
evaluation

3. A. Assume you have 12 food pellets to take to Mars. You must wrap the food pellets in special Mars cloth which costs a dollar a square.

Use 12 cubes to represent the food pellets. The square that costs a dollar is the same size as the square face of the cube.

Describe one way you can stack all 12 food pellets (cubes) in one solid block by giving the edges:

bottom front edge ________

bottom side edge ________

height _______

What does it cost to wrap this package? __________

B. What is most expensive way to wrap the 12 cubes (food pellets)?

edges of the solid block: bottom front ______

bottom side _______

height ______

cost of wrapping ________

C. What is the least expensive way to wrap the 12 cubes (food pellets)?

edges of the solid block: bottom front ______

bottom side _____

height _____

cost of wrapping _______

mouse & elephant
evaluation

4. Do NOT use tiles or cubes. Show all your arithmetic calculations!

What is the area of a rectangle whose dimensions are 13 on the bottom edge and 24 on the side edge? __________ (area)

What is the perimeter of this rectangle?_________

mouse & elephant
evaluation

5. Do not use cubes.

Show arithmetic calculations you use in answering the questions.

The picture below is of a solid block made of cubes.

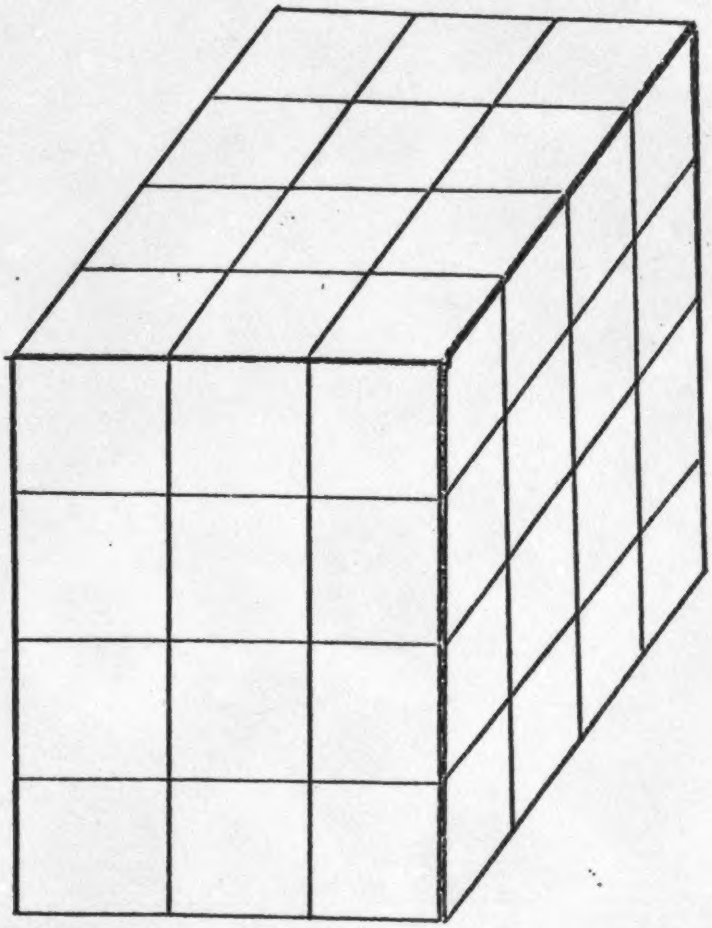
What are the edges? bottom front ________

bottom side ______

height ______

What is the volume? __________

What is the surface area? ________



If you need to ask what volume or surface area mean raise your hand.

mouse & elephant
evaluation

6. How many 1-cubes does it take to make an 8-cube? ____________

How did you figure it out?

How many squares (same size as found on one face of a 1-cube) does it take to cover an 8-cube? ___________

How did you figure it out?

If you need to ask what a 1-cube or an 8-cube are raise your hand.